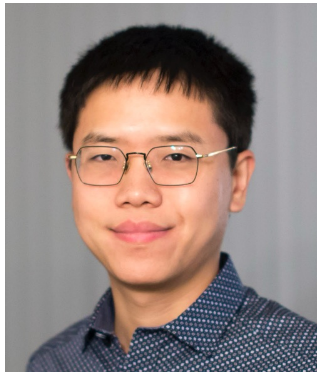
Powers of Tau in Asynchrony


Sourav Das


Zhuolun Xiang


Ling Ren

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Problem Definitions

q-Strong Diffie-Hellman (q-SDH) Assumption

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Challenger

Adversary $\mathcal{A}$

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1. Elliptic curve group $\mathbb{G}$

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1. Elliptic curve group $\mathbb{G}$
2. Scalar field $\mathbb{F}$

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q-Strong Diffie-Hellman (q-SDH) Assumption

1. Elliptic curve group $\mathbb{G}$
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3. Generator $\mathbb{G} \in \mathbb{G}$

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$$\xleftarrow{\quad q \in \mathbb{N} \quad}$$

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$\tau \leftarrow \mathbb{F}$

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$q \in \mathbb{N}$ ($\leftarrow$)

$\mathrm{G}, \tau\mathrm{G}, \tau^2\mathrm{G}, \ldots, \tau^q\mathrm{G}$ ($\rightarrow$)

$c, (\tau + c)^{-1} \cdot \mathrm{G}$ ($\leftarrow$)

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Adversary wins: if $c + \tau \neq 0$ and $(\tau + c)^{-1} \cdot \mathrm{G}$ is well formed

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$q \in \mathbb{N}$ (Adversary → Challenger)

$\mathrm{G}, \tau\mathrm{G}, \tau^2\mathrm{G}, \ldots, \tau^q\mathrm{G}$ (Challenger → Adversary)

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q-SDH assumptions says adversary wins with negligible probability

q-SDH parameters (aka Powers of Tau)

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Applications of q-SDH parameters

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- Short signatures

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- Cryptographic Accumulators

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  - Randomness Beacon

Our Goal

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MPC protocol to generate of Powers of Tau in an asynchronous network

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System model:

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- $n \geq 3t + 1$ nodes among which up to t are corrupt

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MPC protocol to generate of Powers of Tau in an asynchronous network

System model:

- $n \geq 3t + 1$ nodes among which up to t are corrupt
- Asynchronous network:
  - Message delays could be arbitrary

Related Works

Generic MPC based approach

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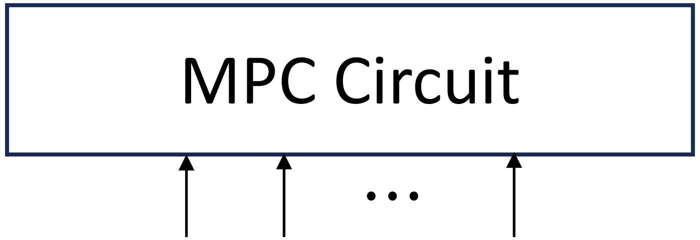

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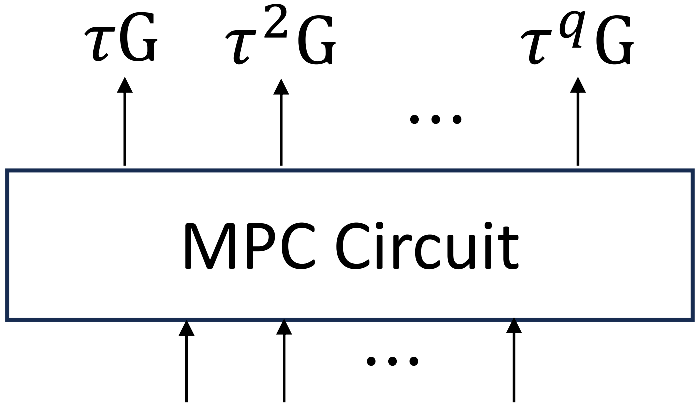

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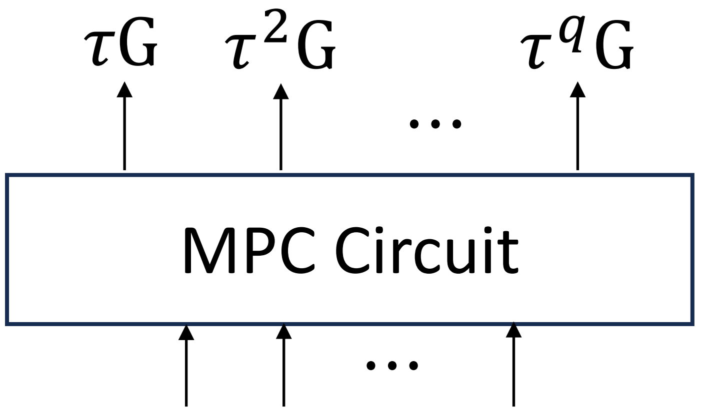

- MPC over both field $\mathbb{F}$ and group $\mathbb{G}$

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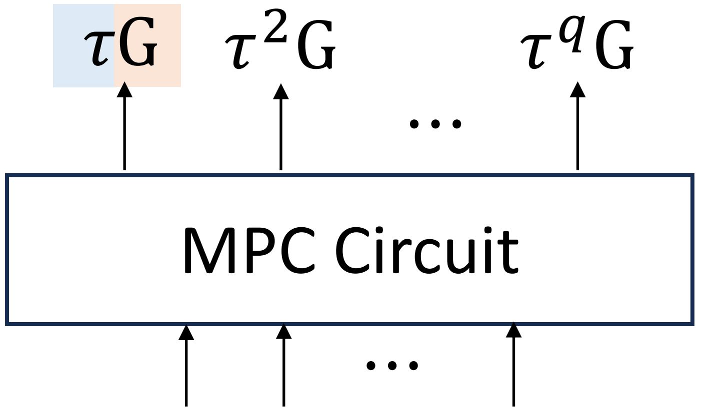

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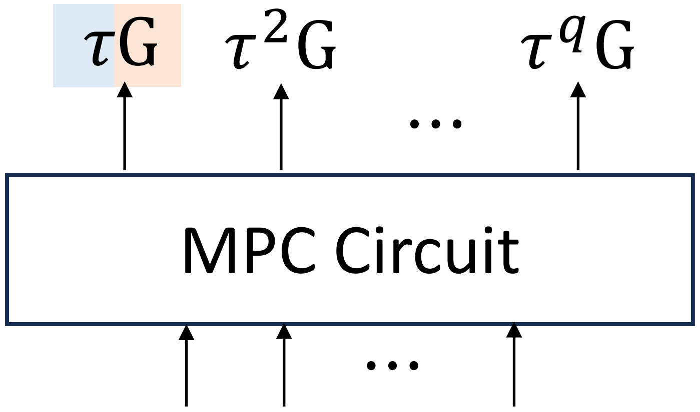

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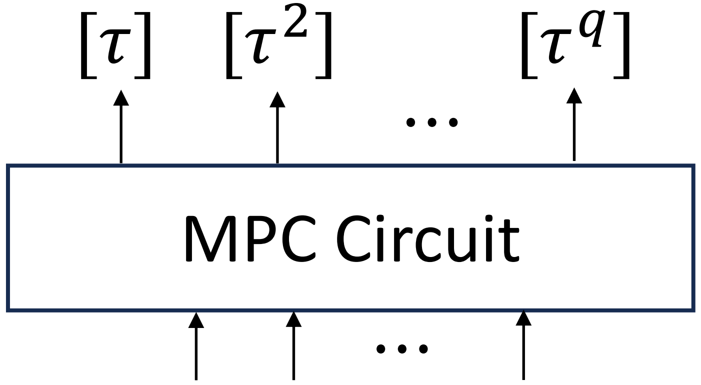

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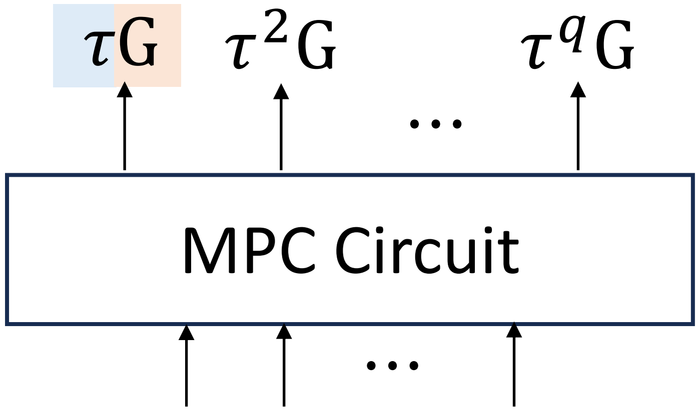

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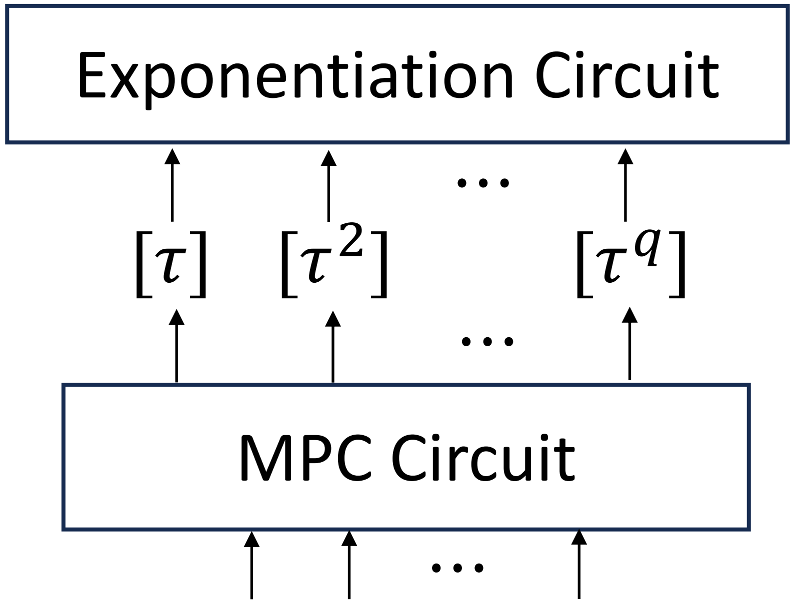

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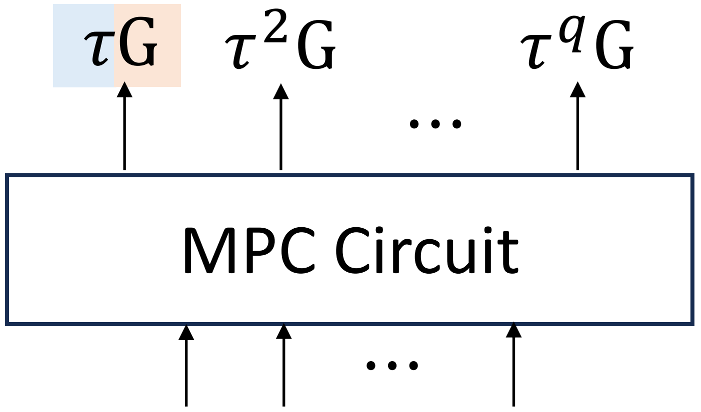

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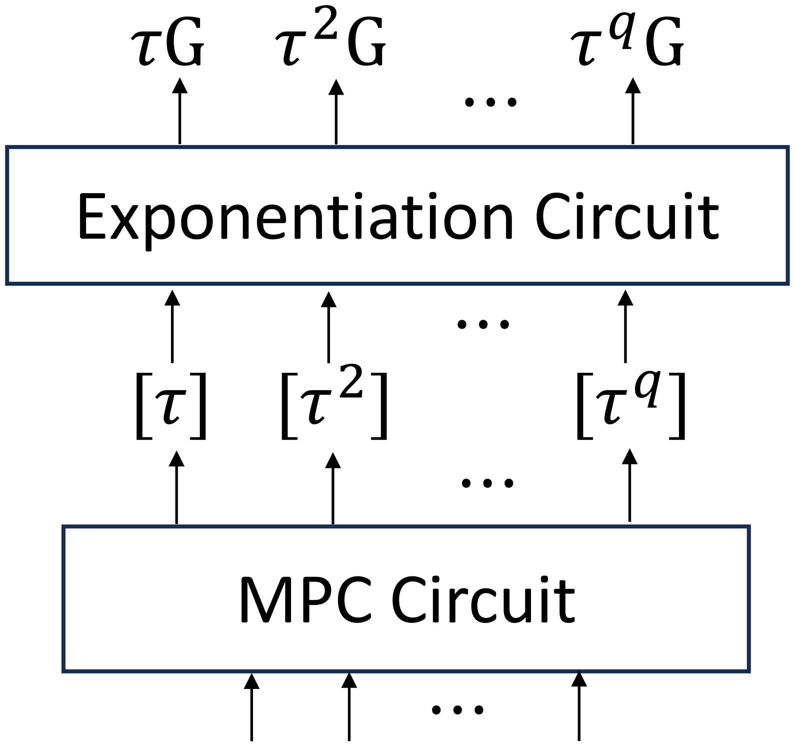

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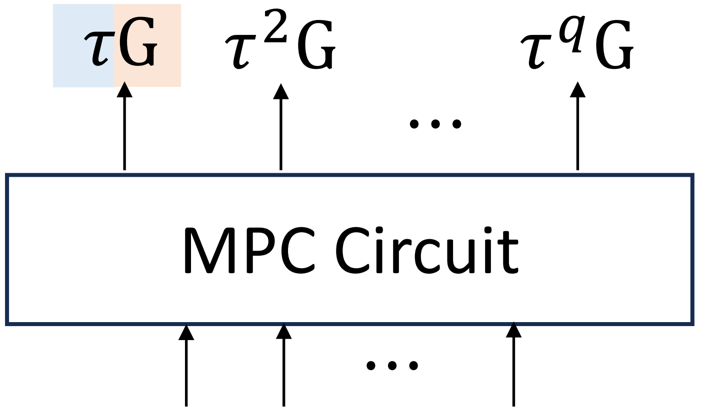

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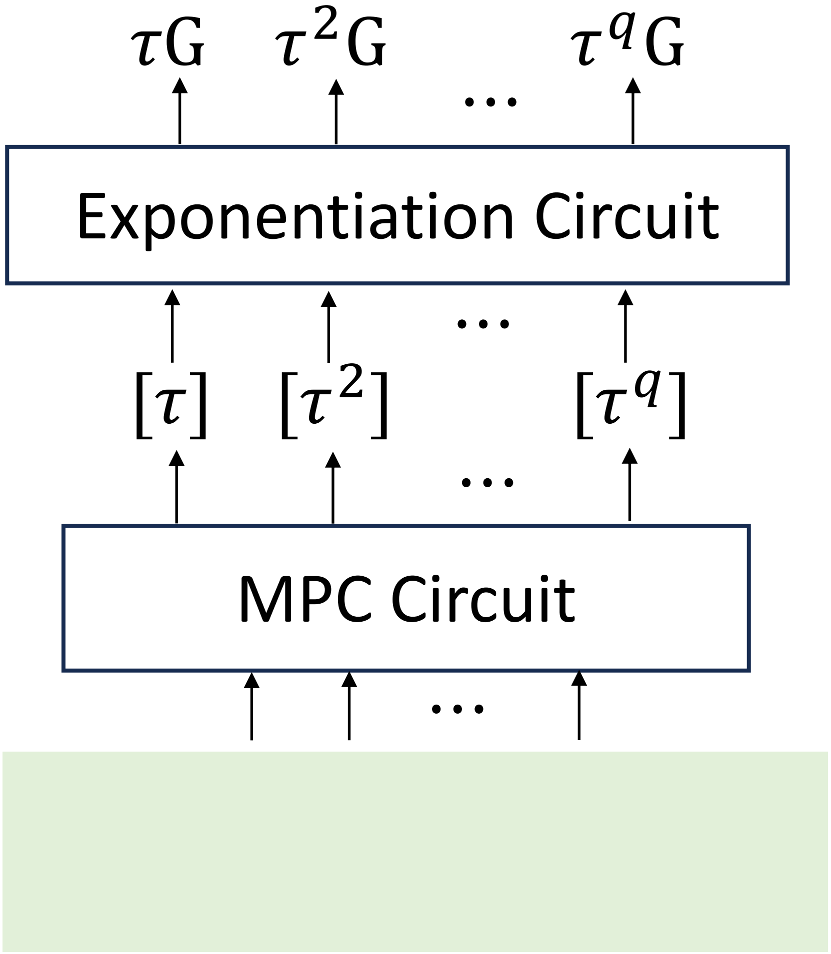

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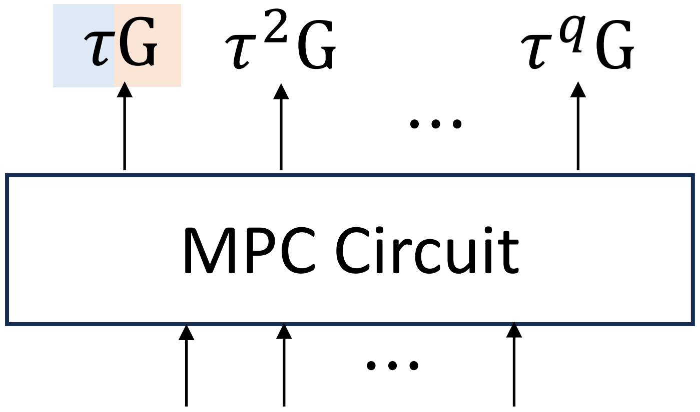

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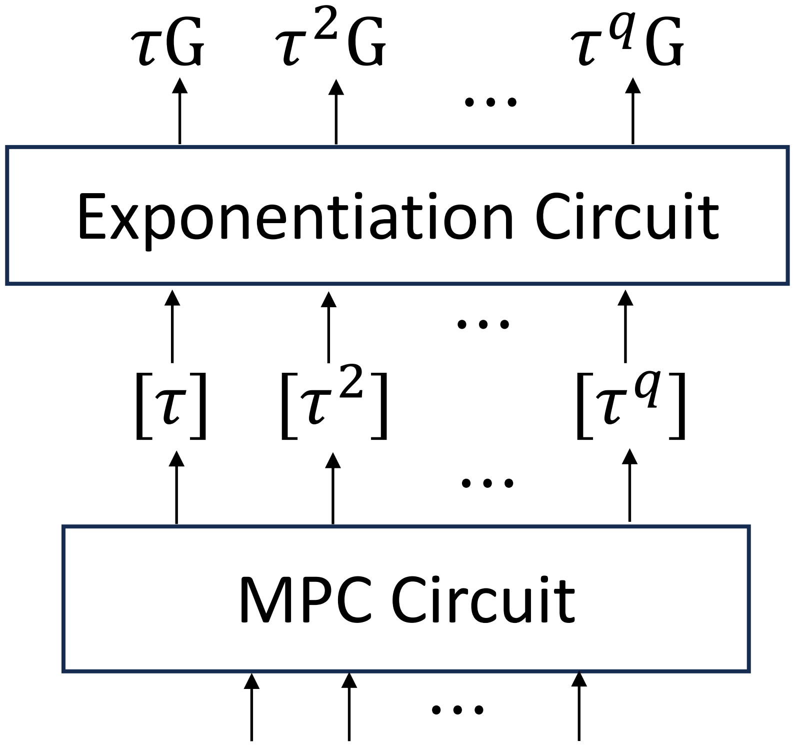

+ MPC over only field $\mathbb{F}$

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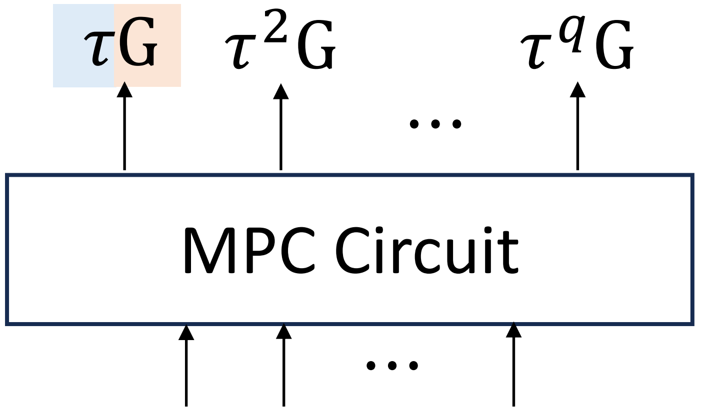

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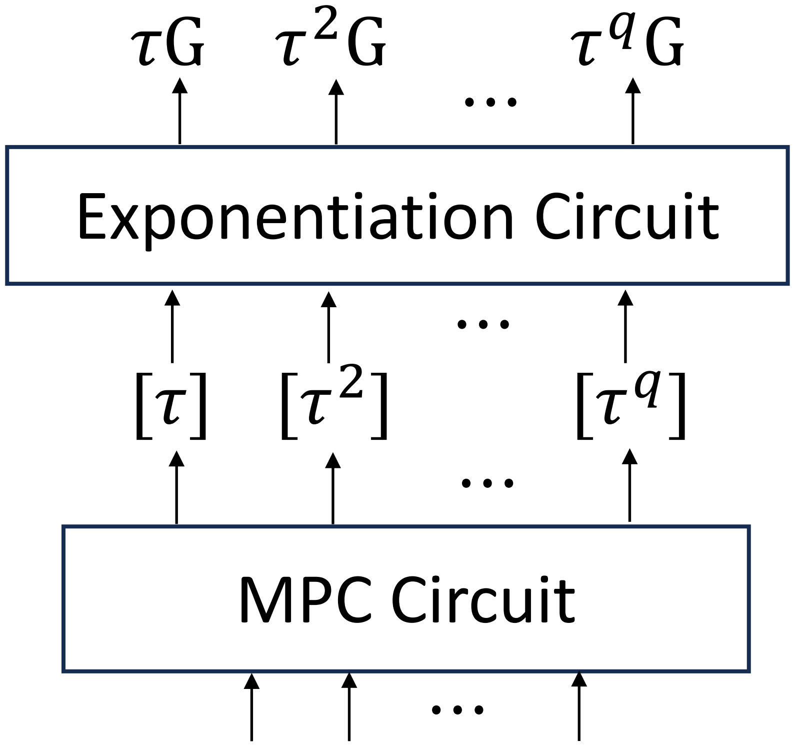
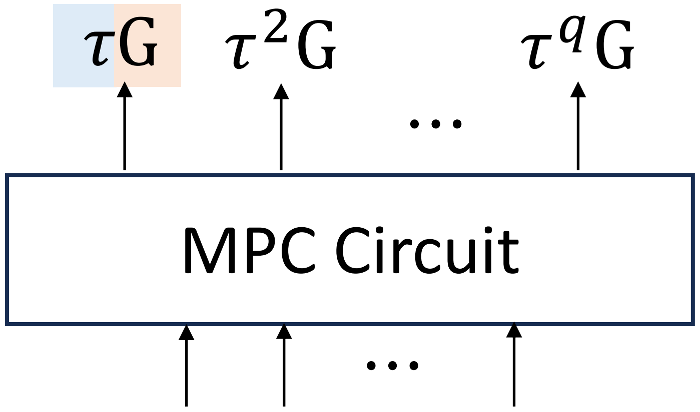


+ MPC over only field $\mathbb{F}$
+ $O(\log q)$ rounds

Generic MPC based approach



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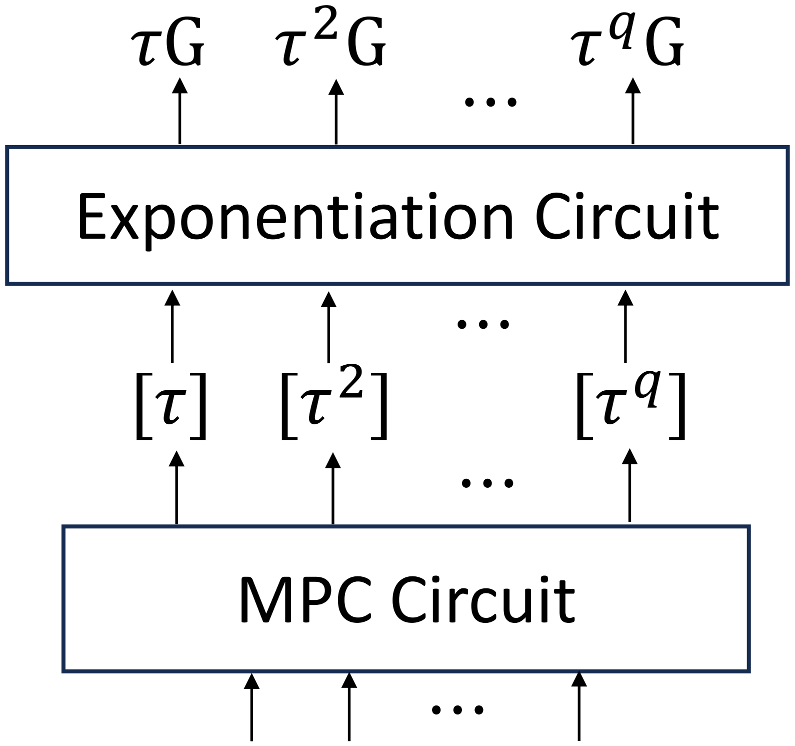

+ MPC over only field $\mathbb{F}$
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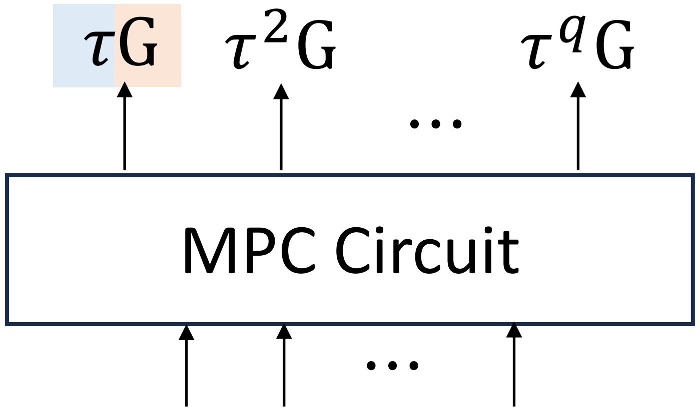

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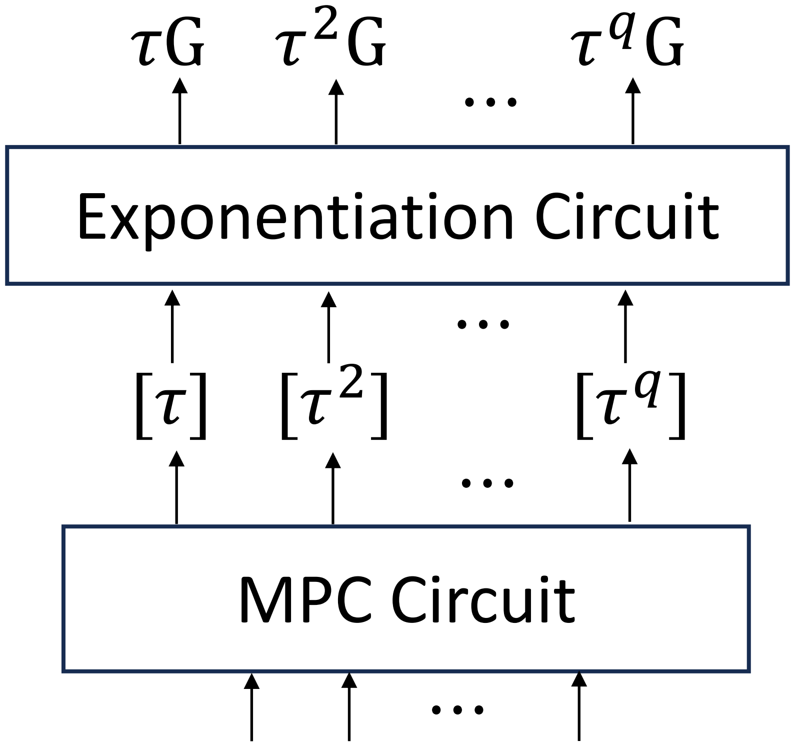

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Multiplication units are expensive, per party $\Omega(nq)$ communication costs

Round Robin approach

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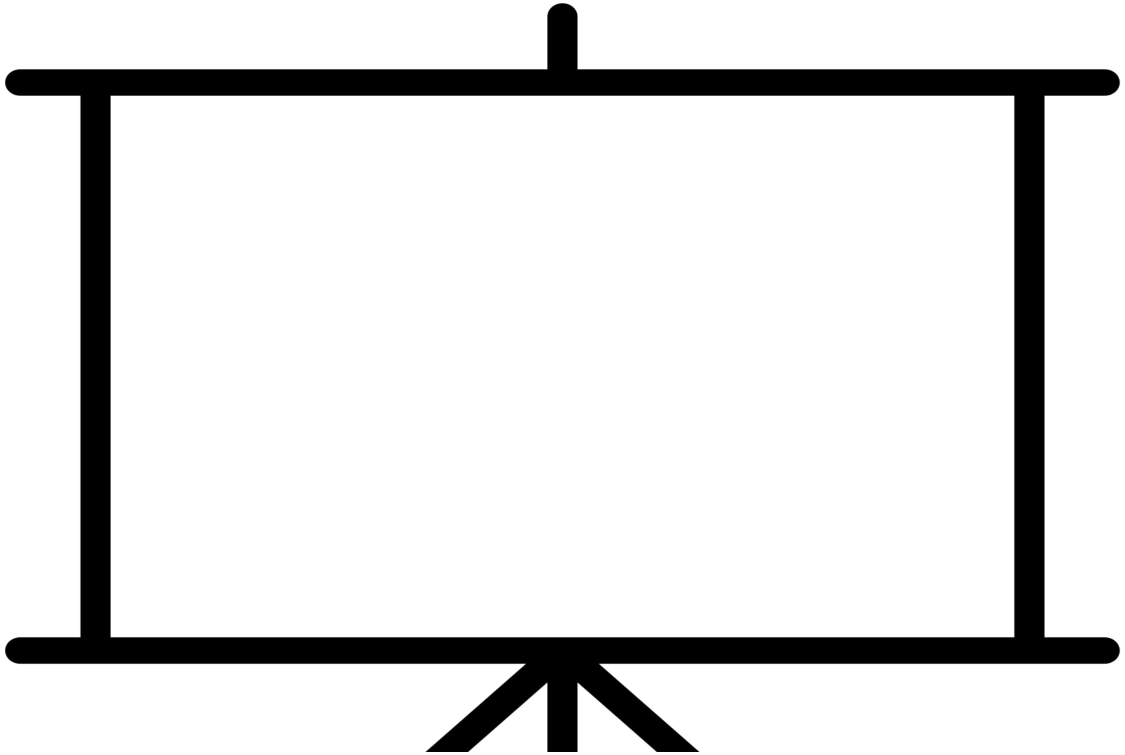

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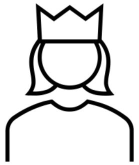

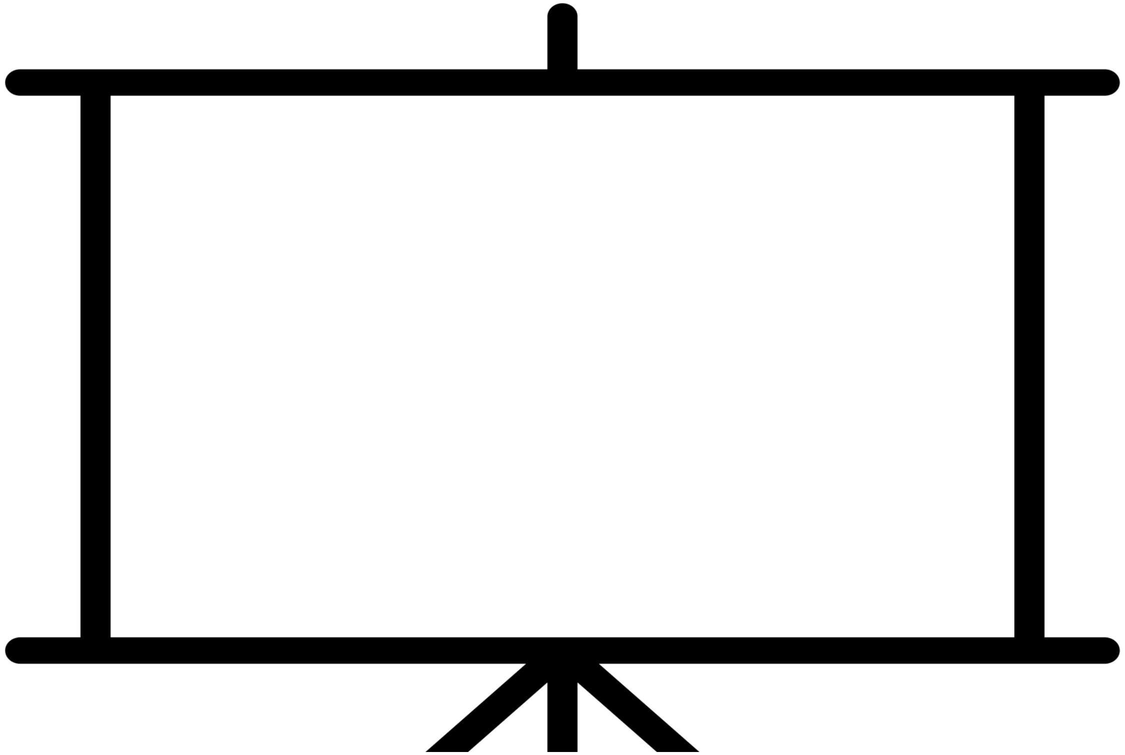

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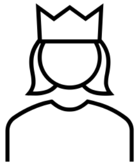

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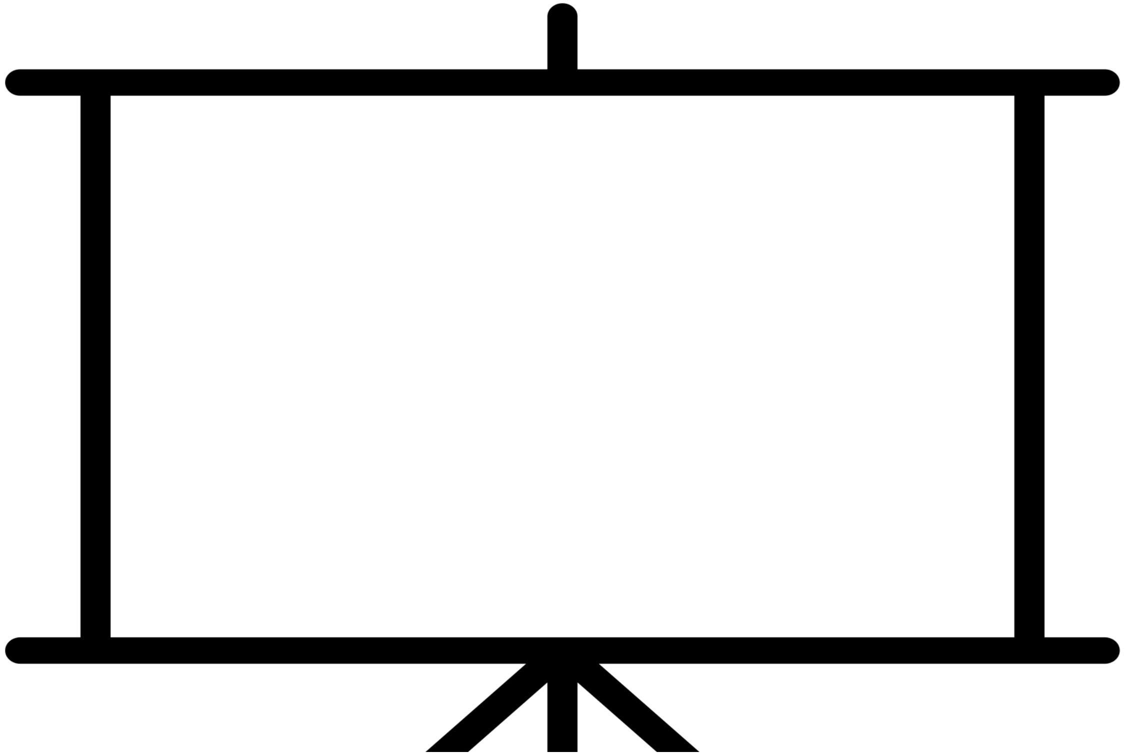

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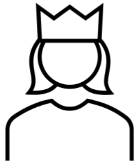

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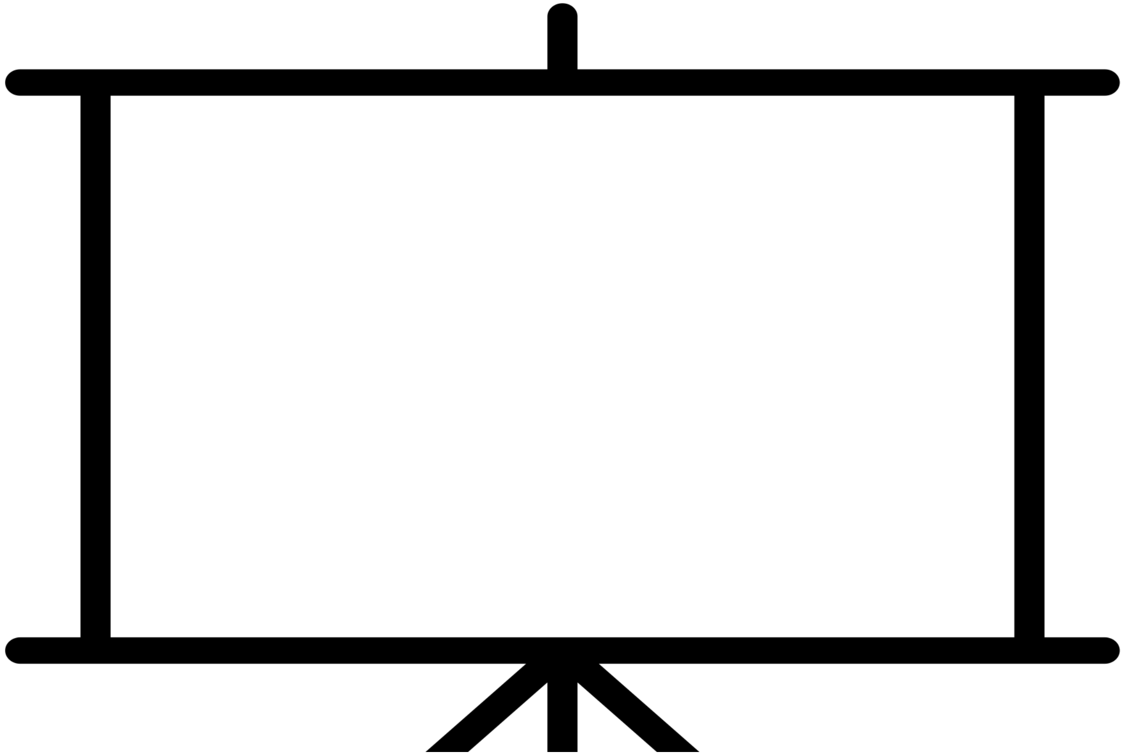

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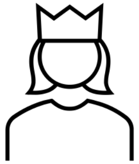

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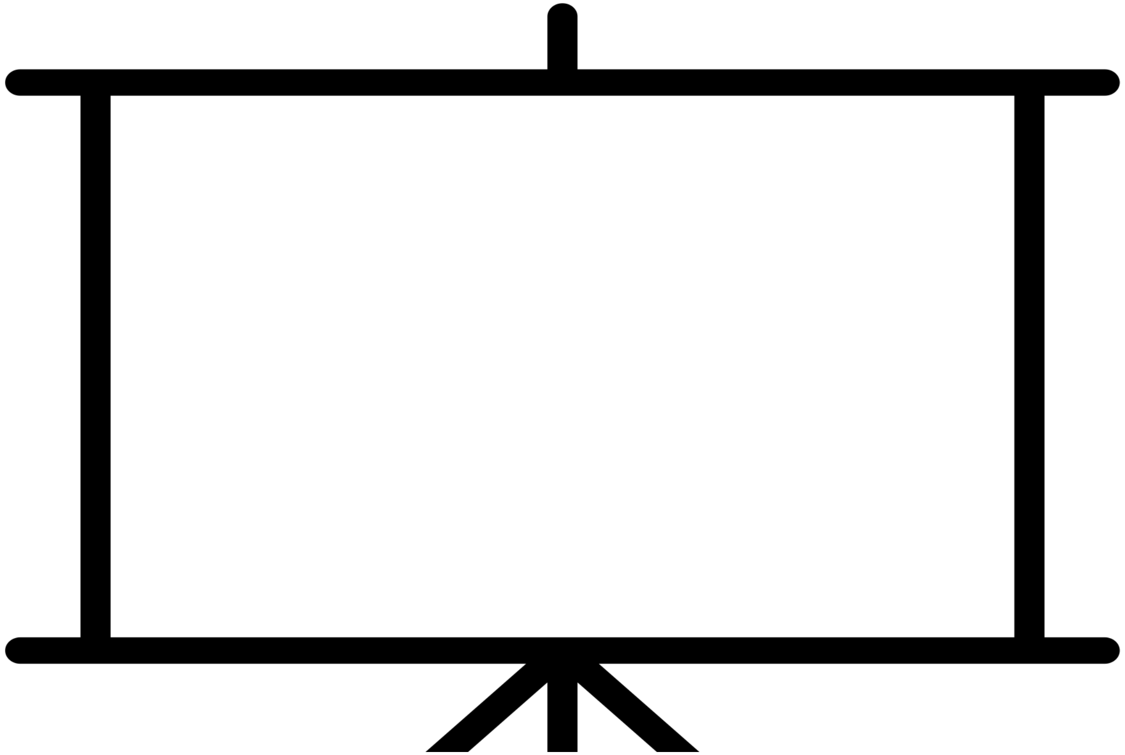

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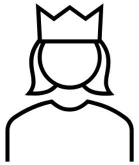

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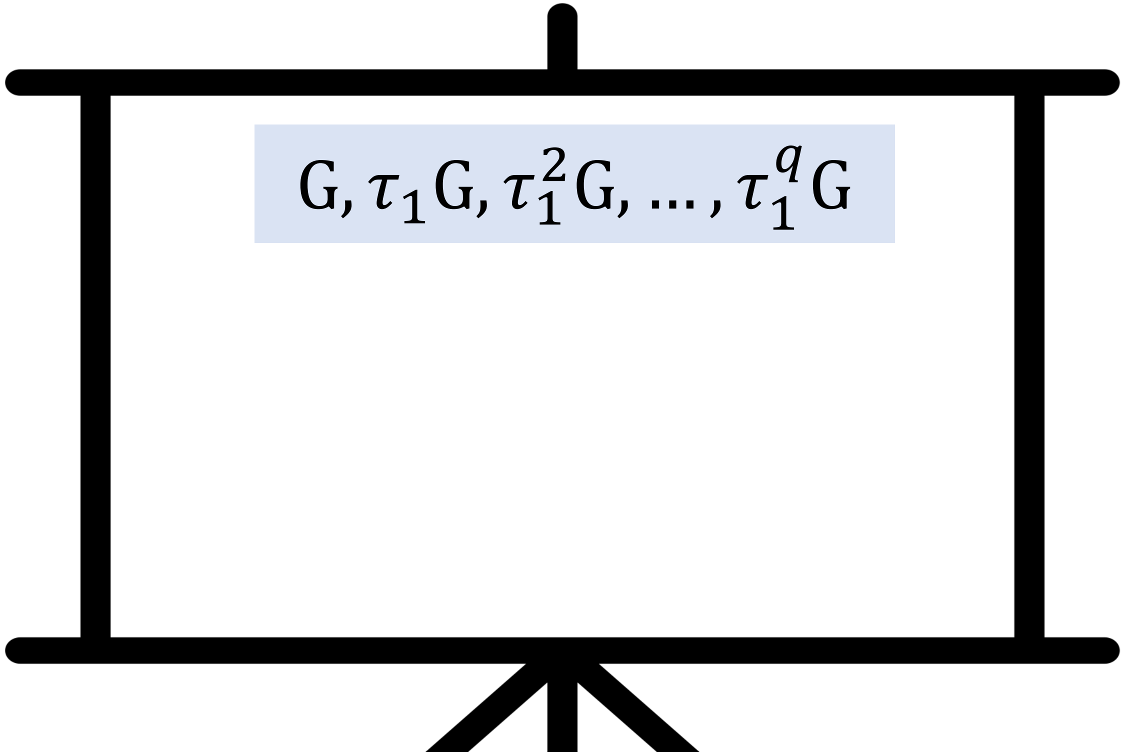

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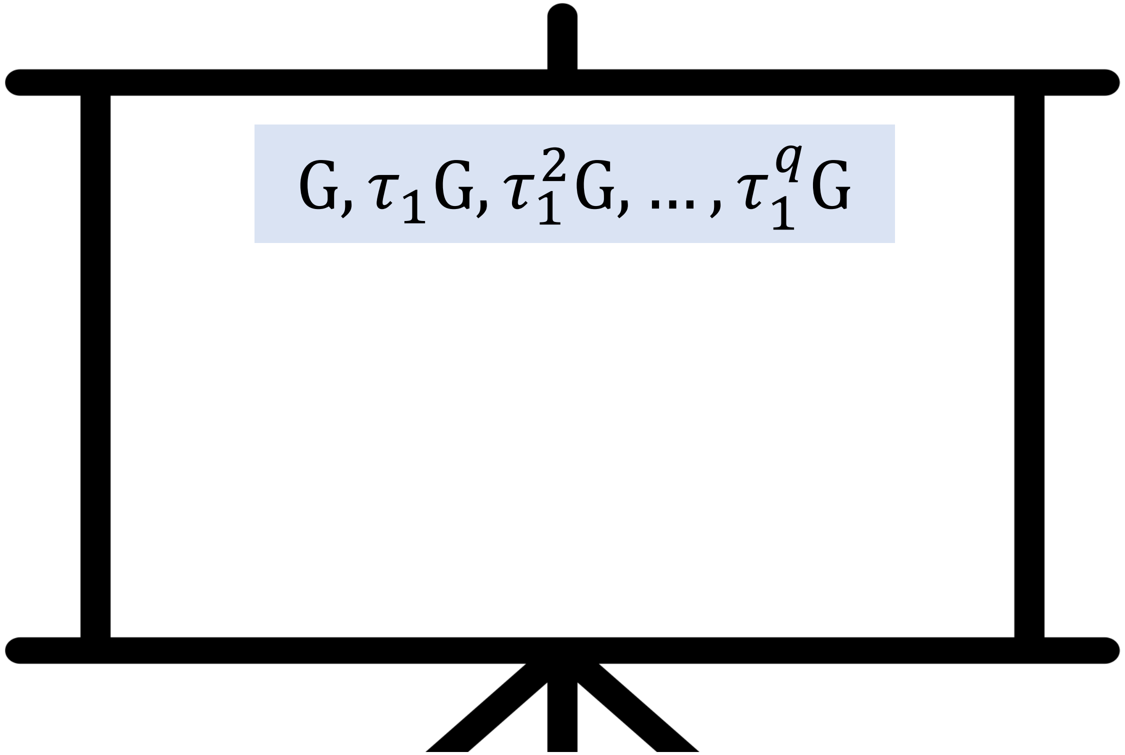

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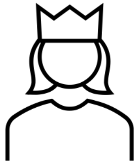

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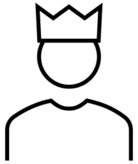
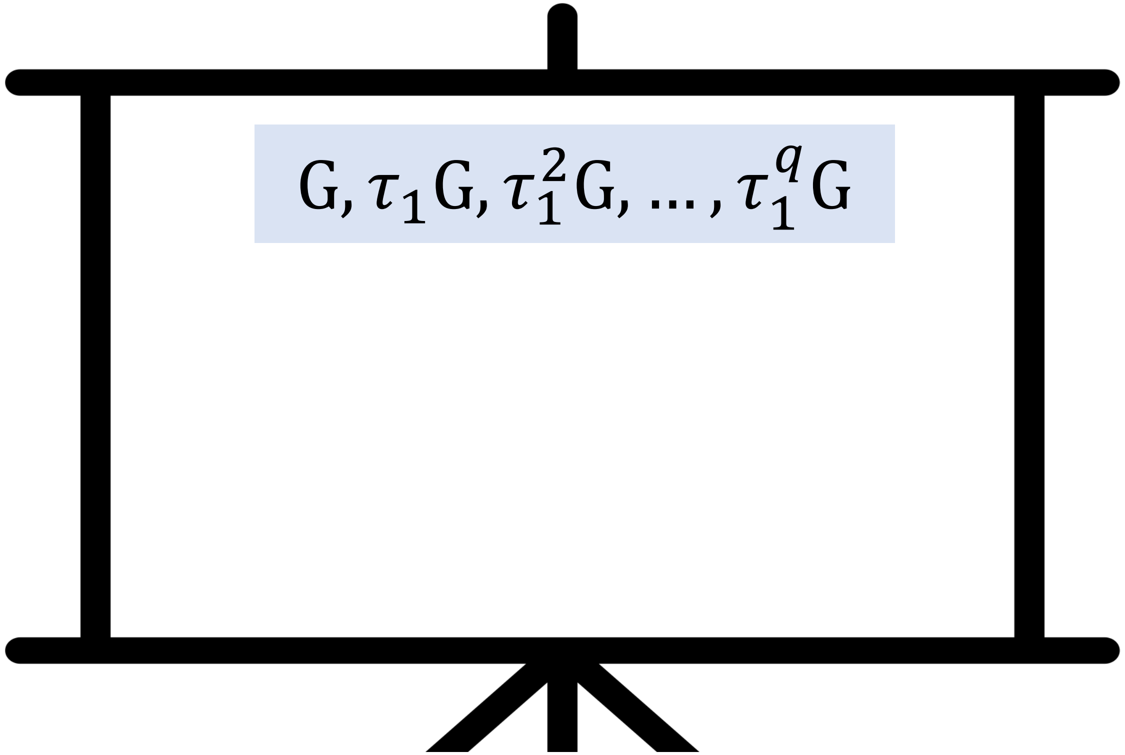


- Download $\mathrm{G}_0, \mathrm{G}_1, \mathrm{G}_2, \ldots, \mathrm{G}_q$



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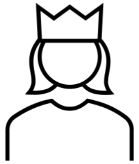

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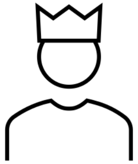

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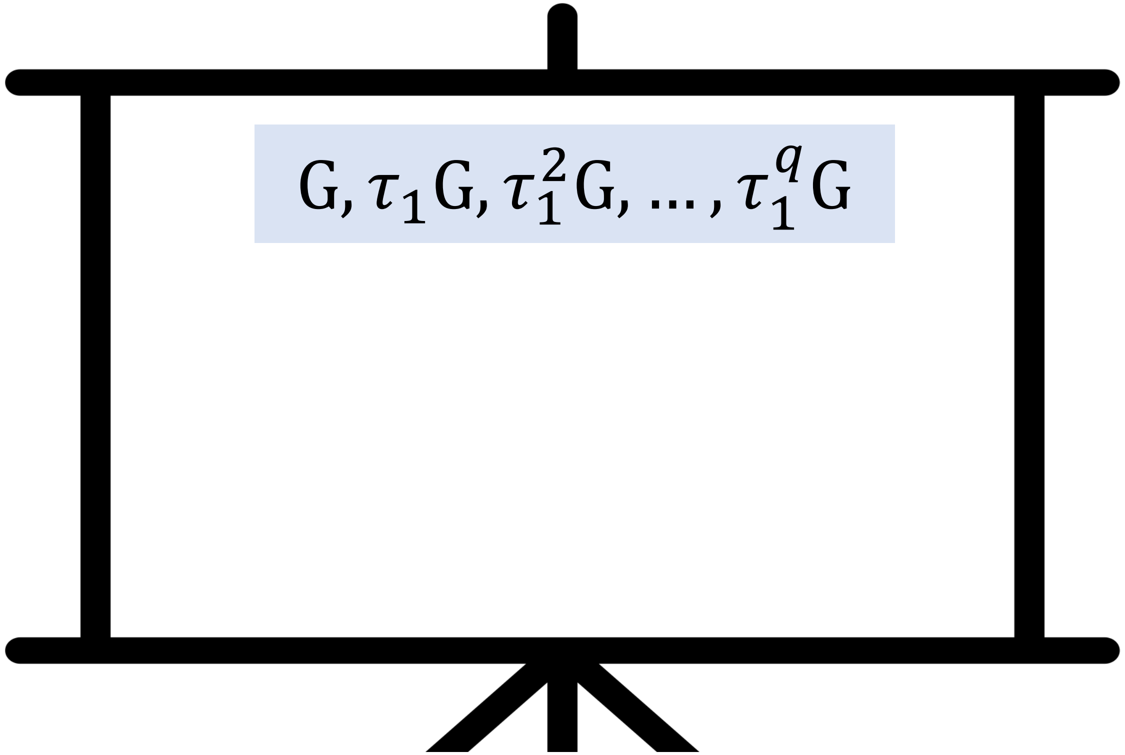

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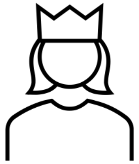

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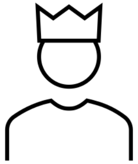

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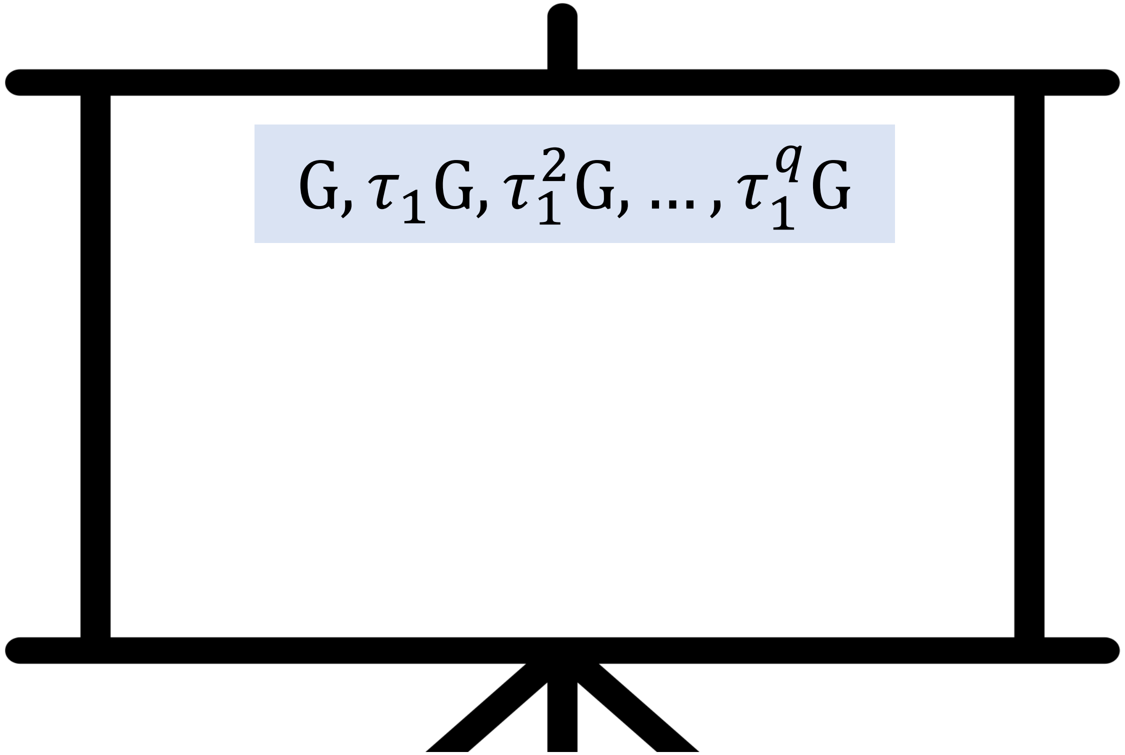

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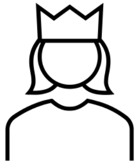

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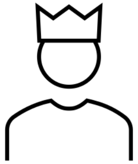

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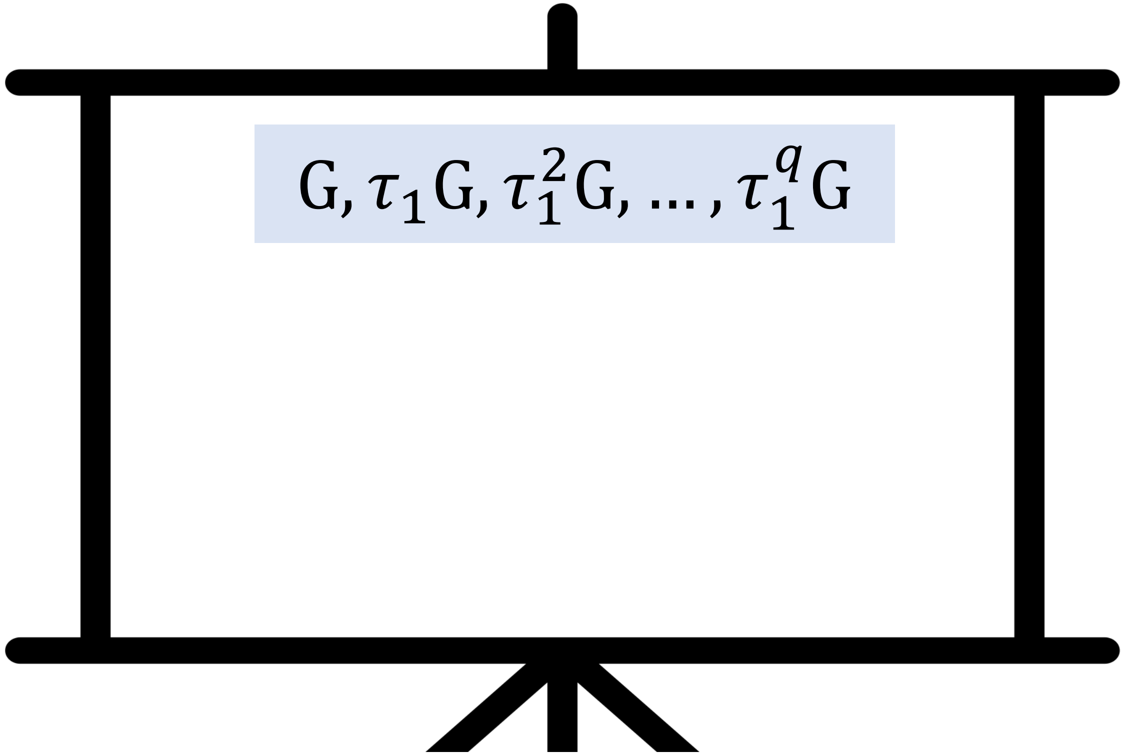
- Compute $\mathrm{G}_0, \tau_2\mathrm{G}_1, \tau_2^2\mathrm{G}_2, \ldots, \tau_2^q\mathrm{G}_q$
- Broadcast $\mathrm{G}_0, \tau_2\mathrm{G}_1, \tau_2^2\mathrm{G}_2, \ldots, \tau_2^q\mathrm{G}_q$



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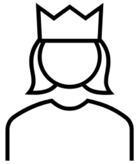

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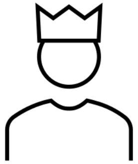

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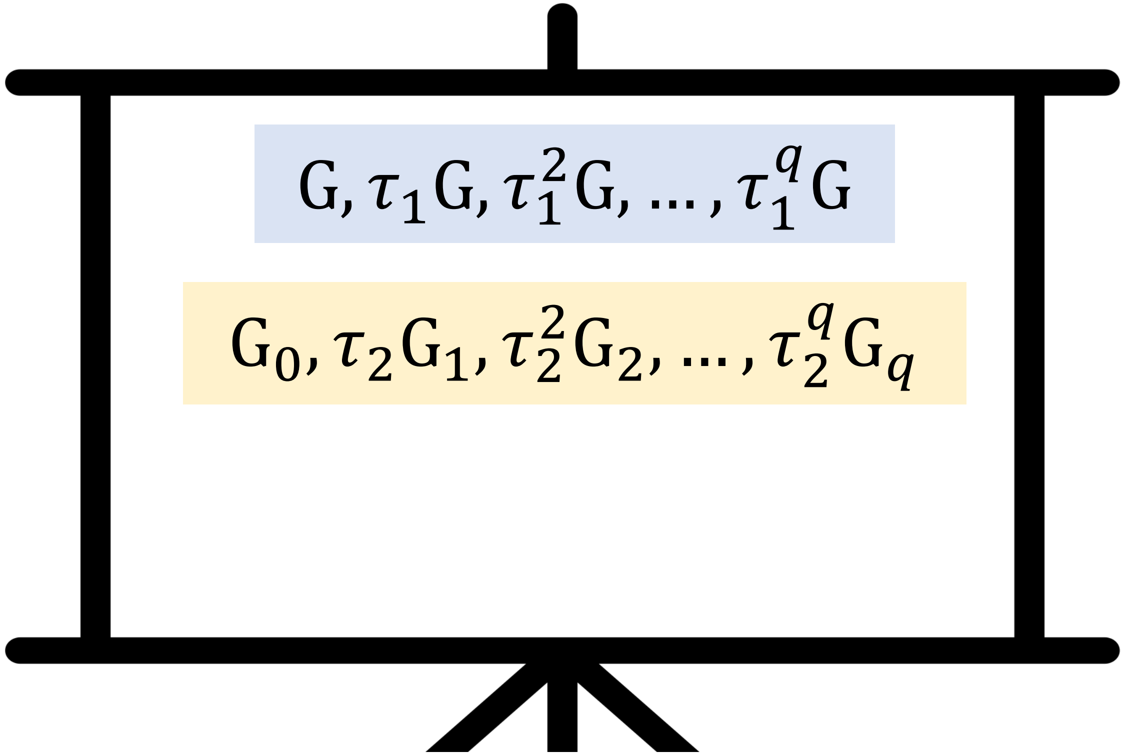

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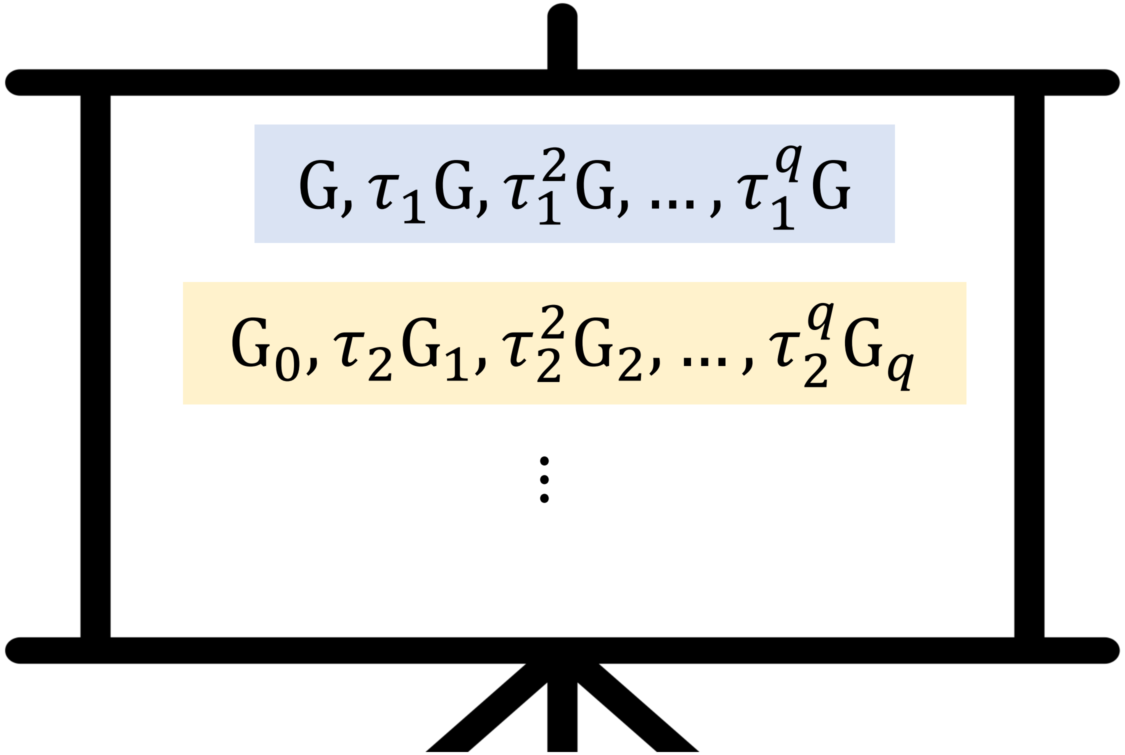

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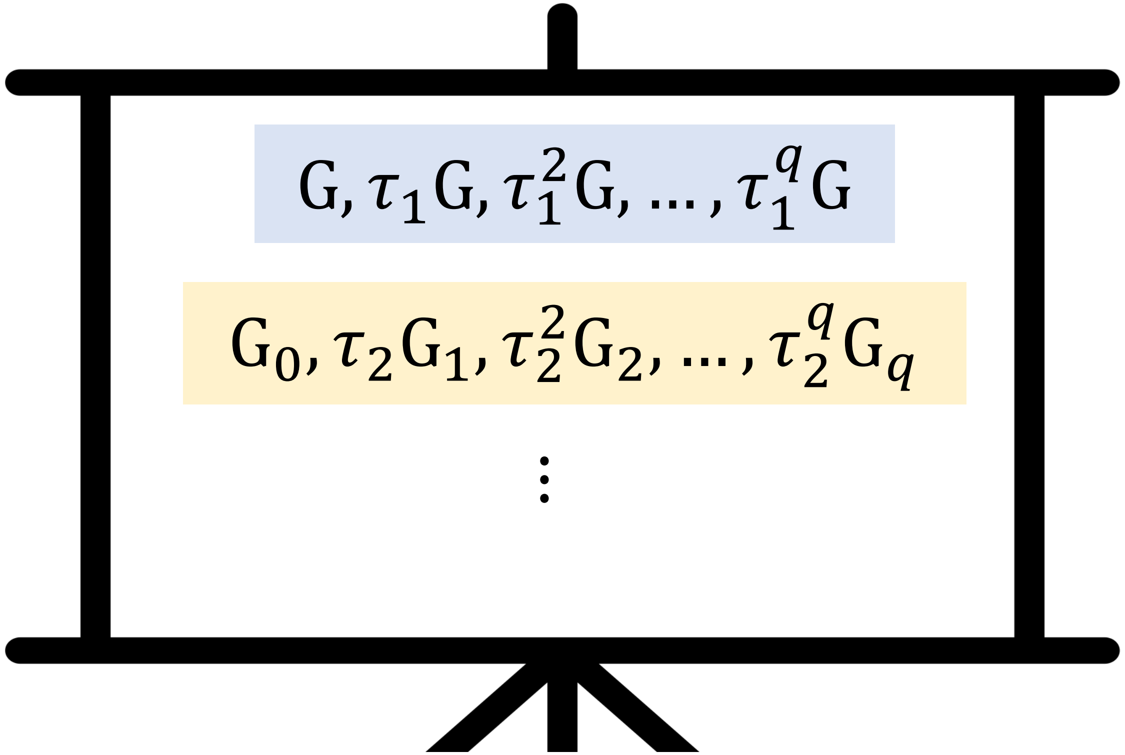

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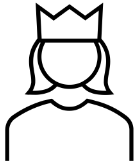

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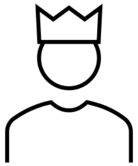

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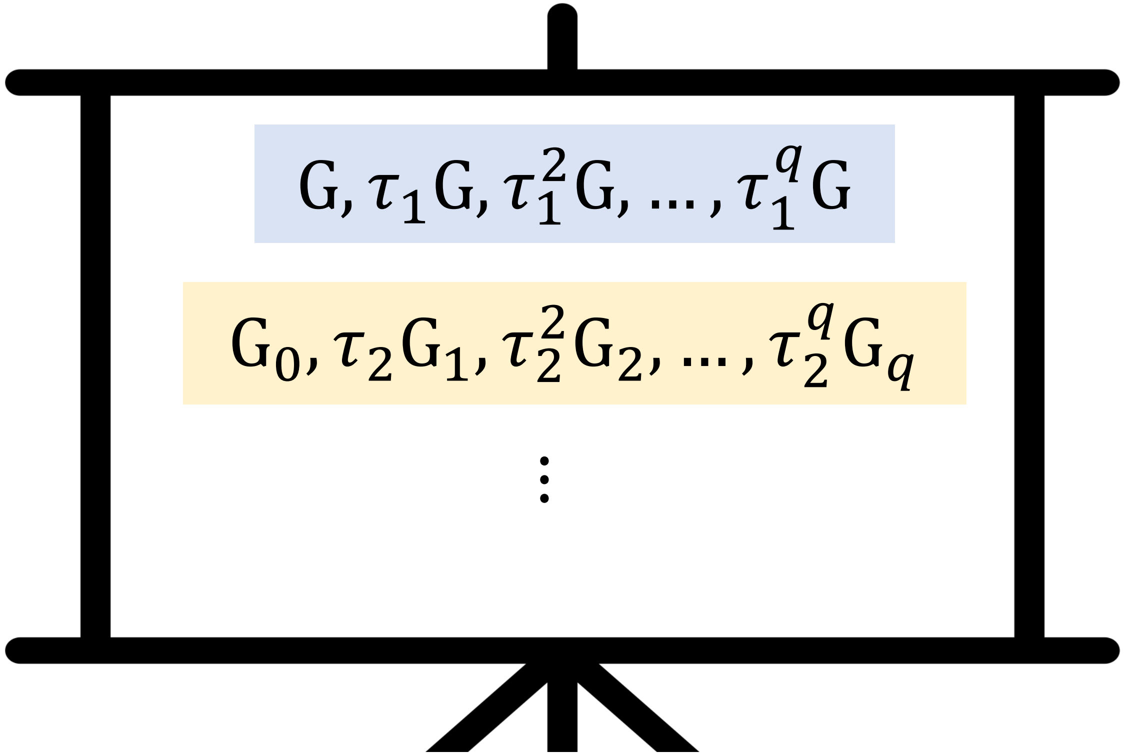

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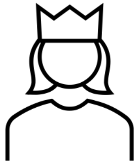

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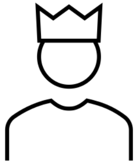

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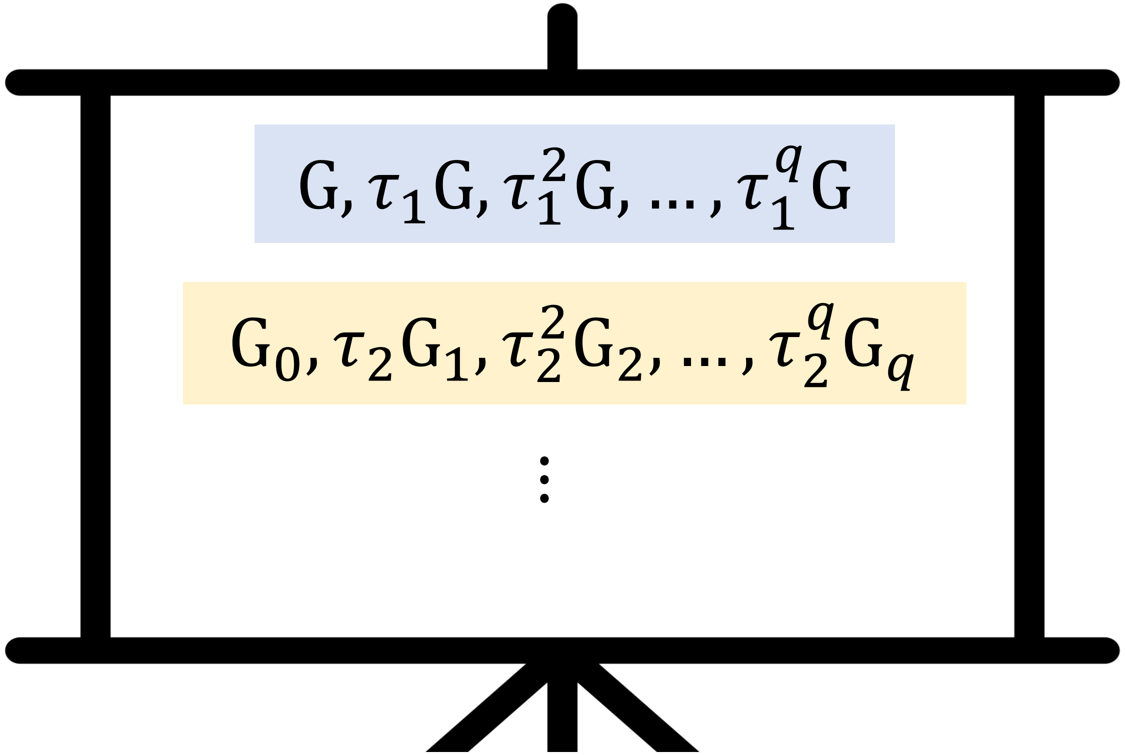

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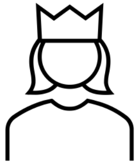

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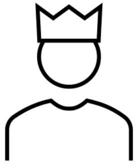

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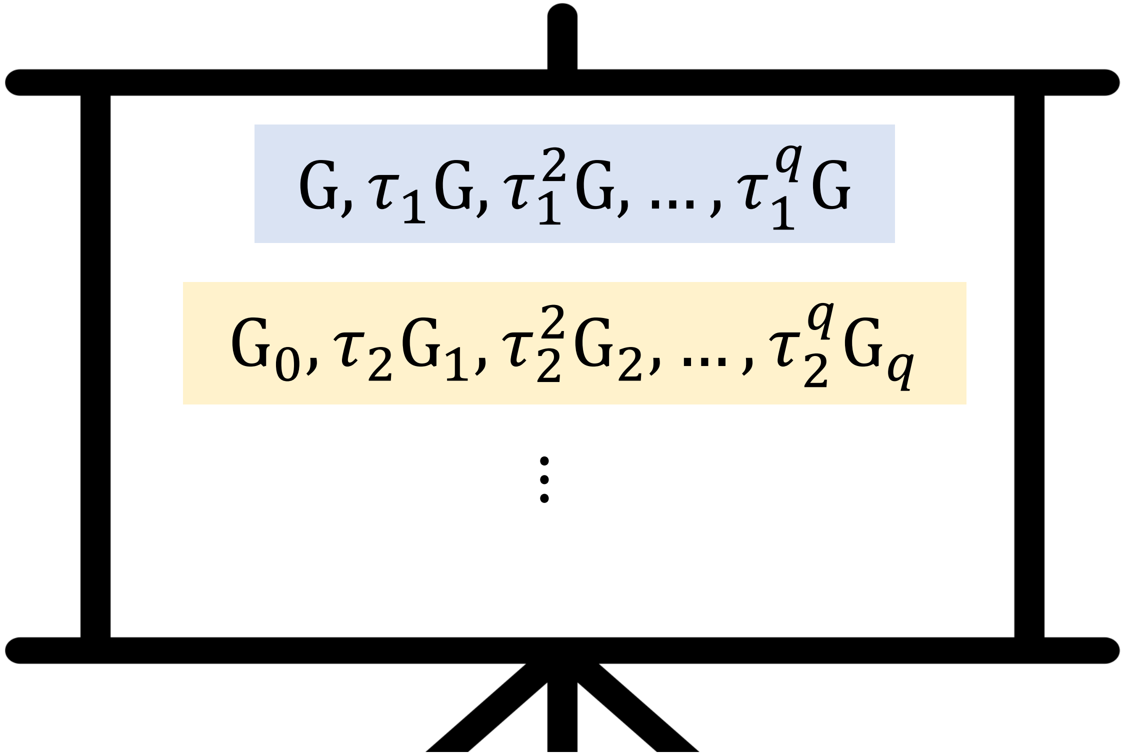

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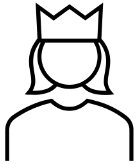

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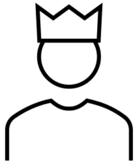

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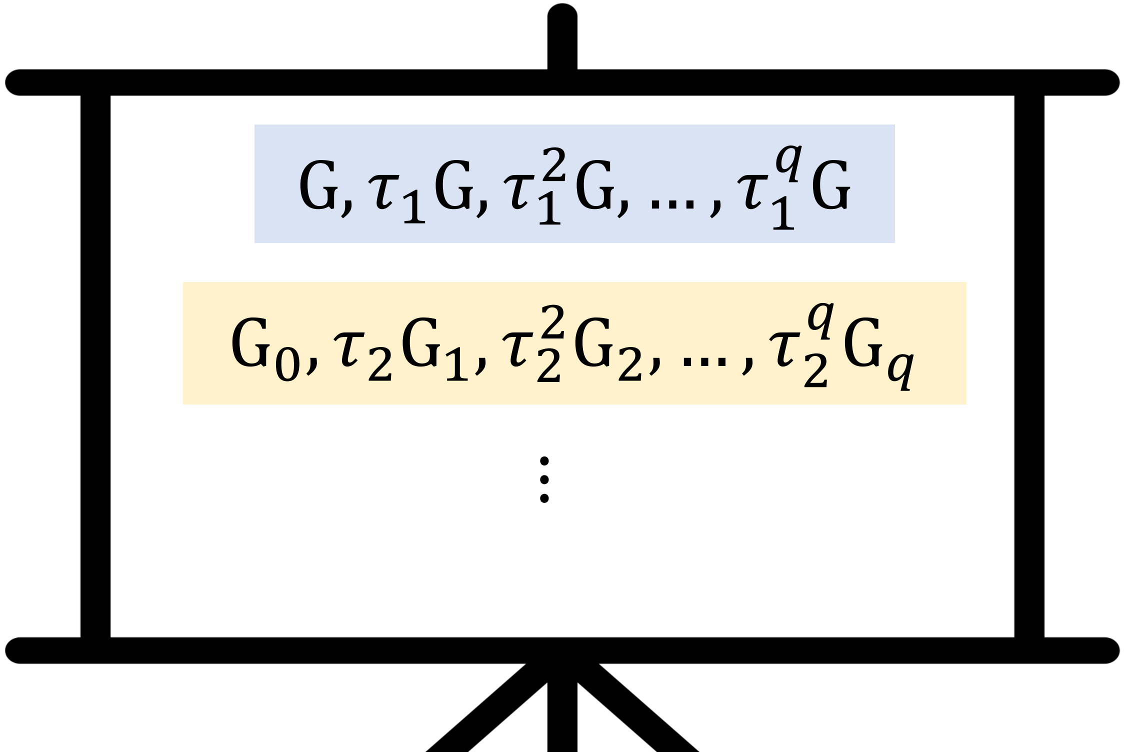

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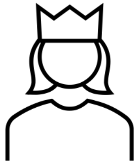

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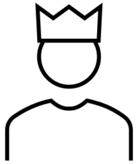

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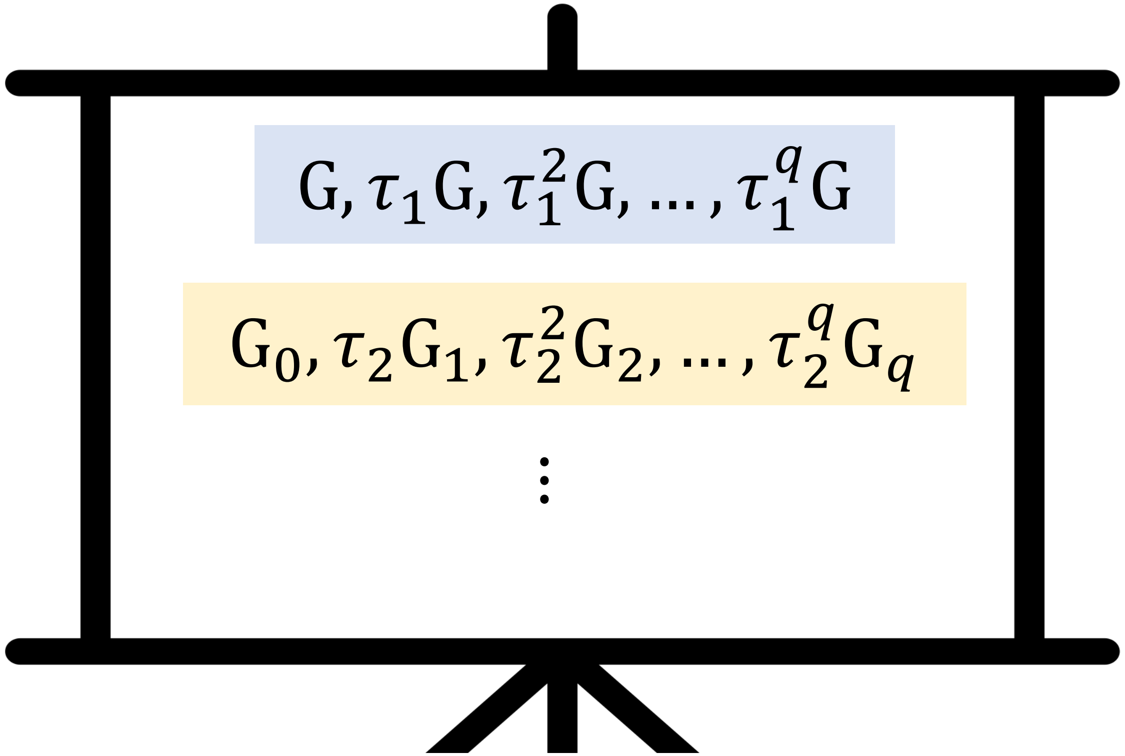

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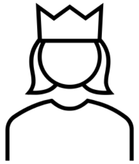

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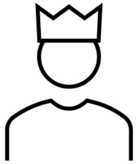

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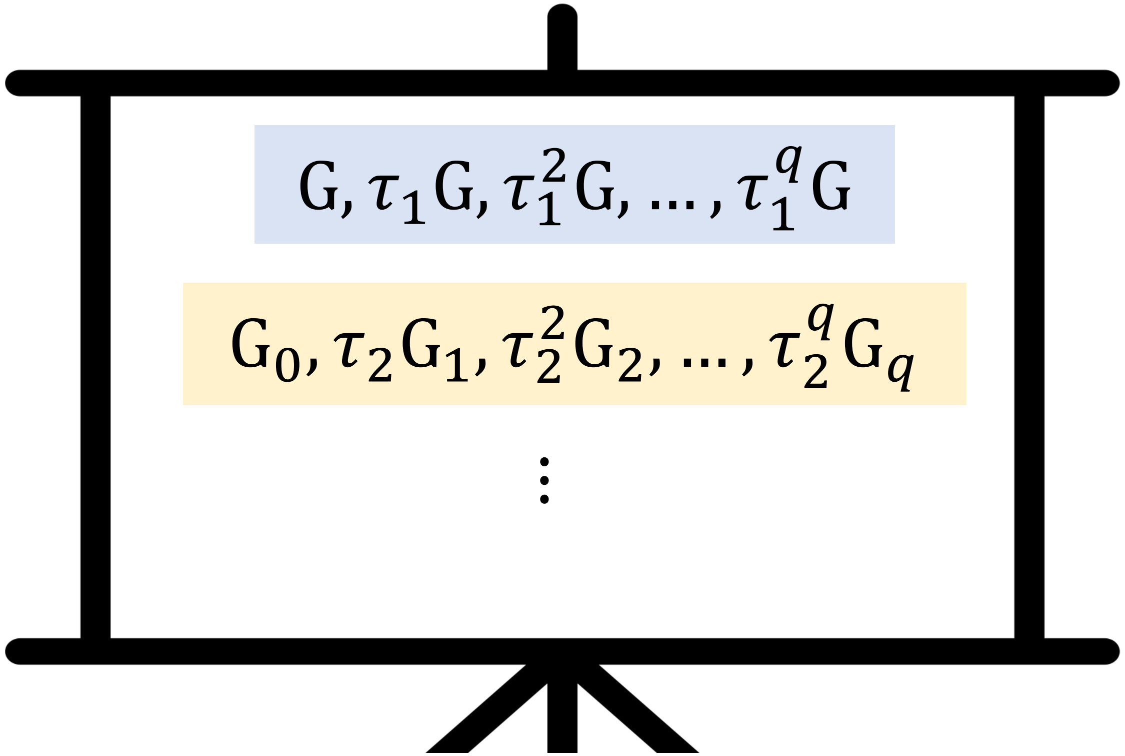

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- Does not work in asynchrony

Our Approach

Our Approach: High-level

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Specialized asynchronous MPC for generating Powers of Tau

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Three phases:

Our Approach: High-level

Specialized asynchronous MPC for generating Powers of Tau

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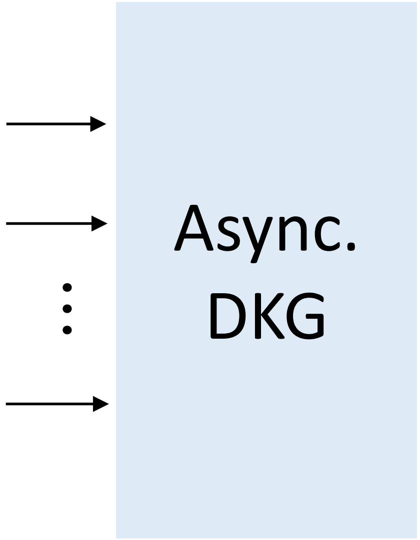

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Specialized asynchronous MPC for generating Powers of Tau

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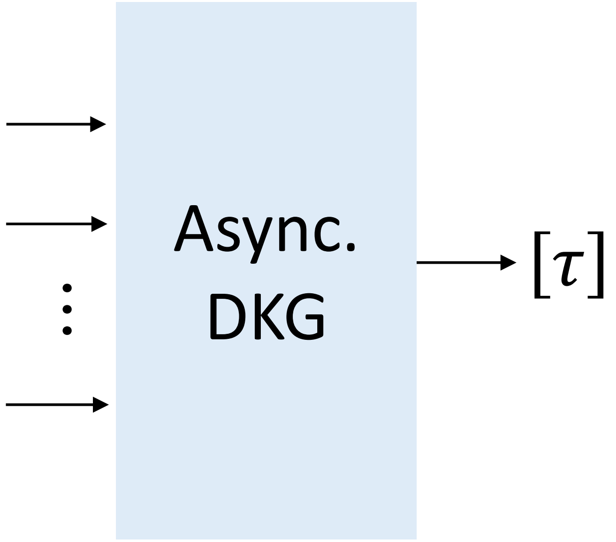

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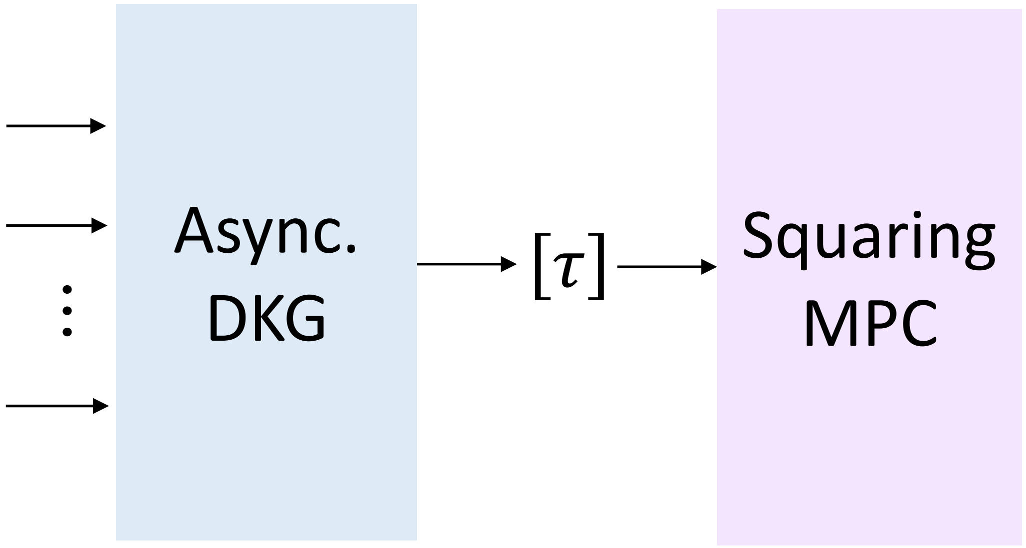

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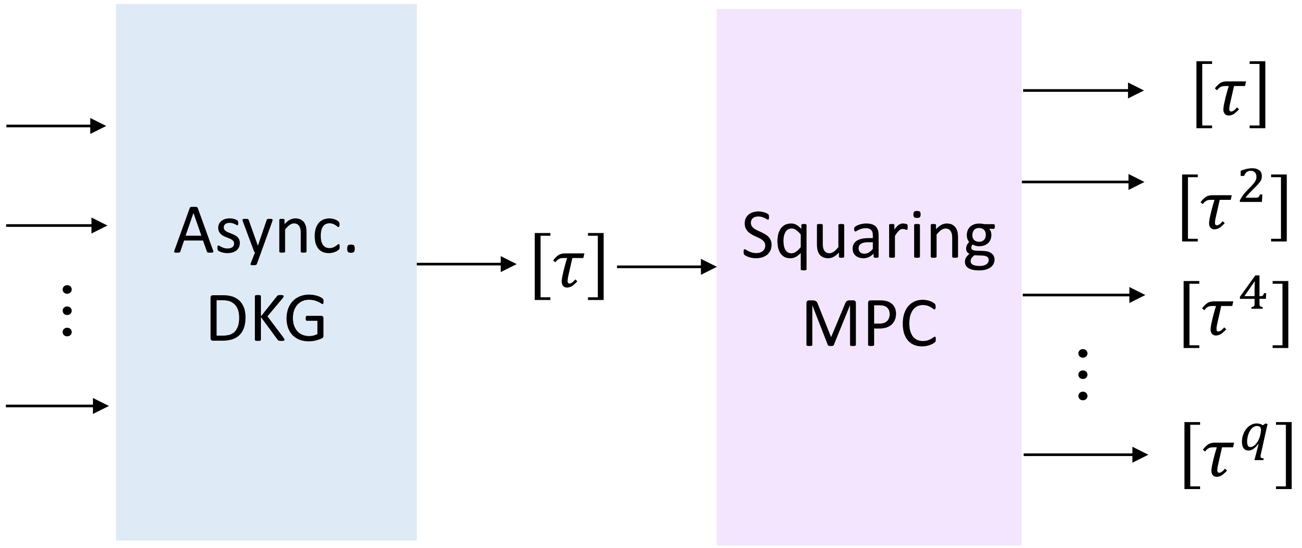

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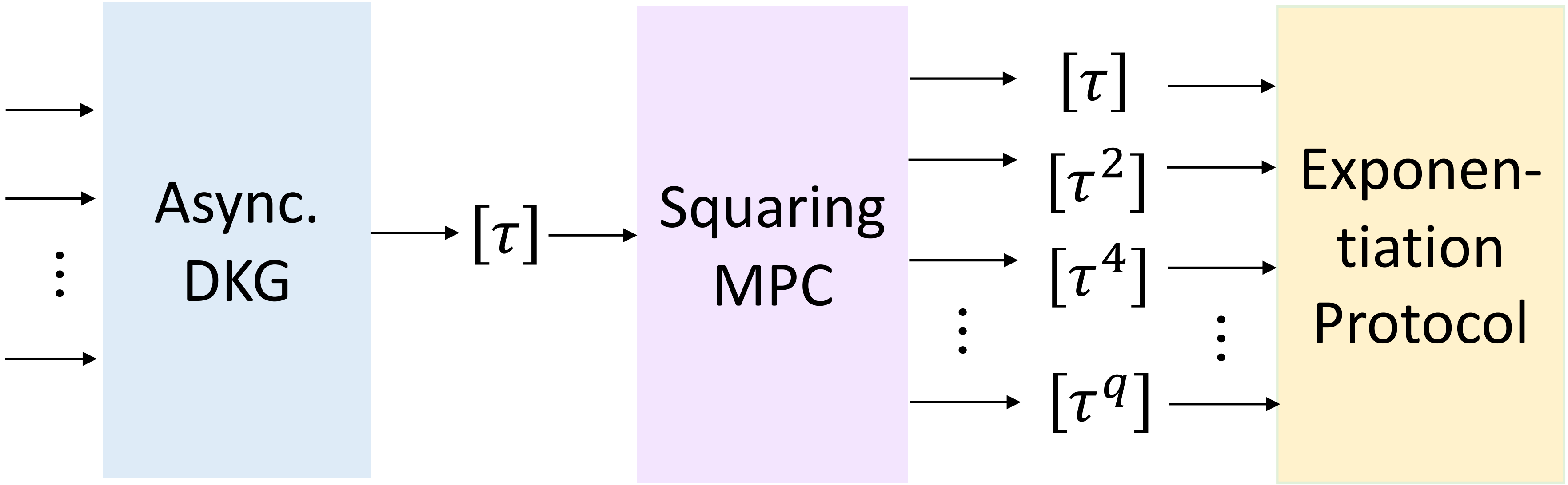

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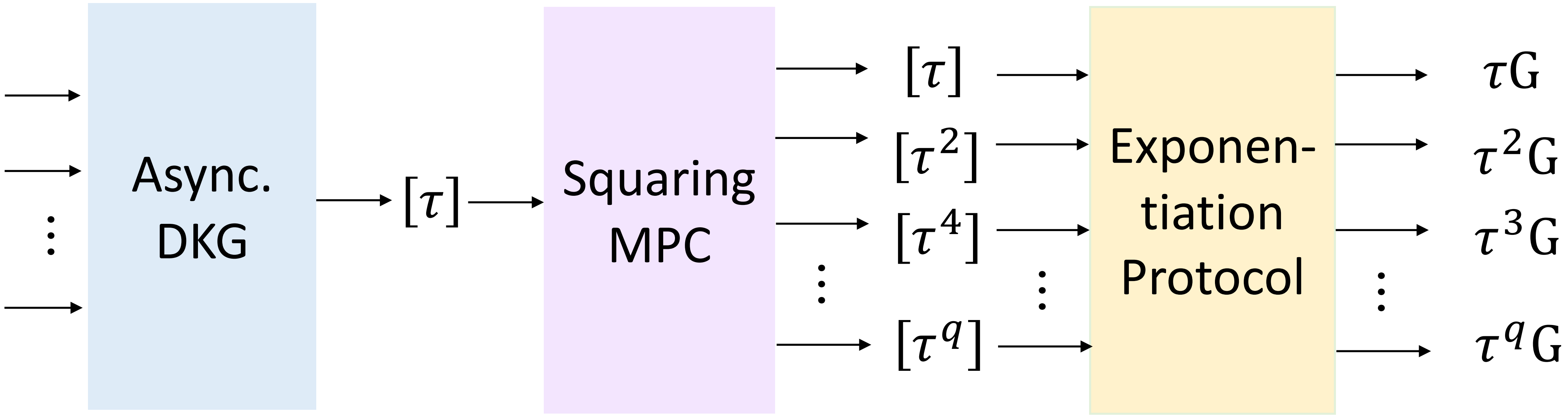

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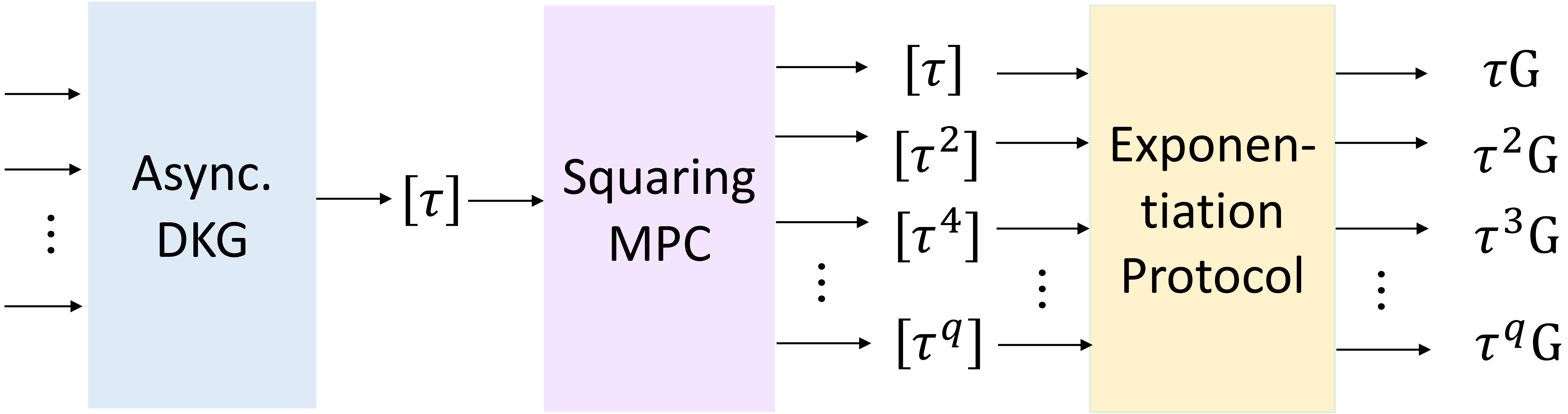

Only $O(\log q)$ multiplication units are needed

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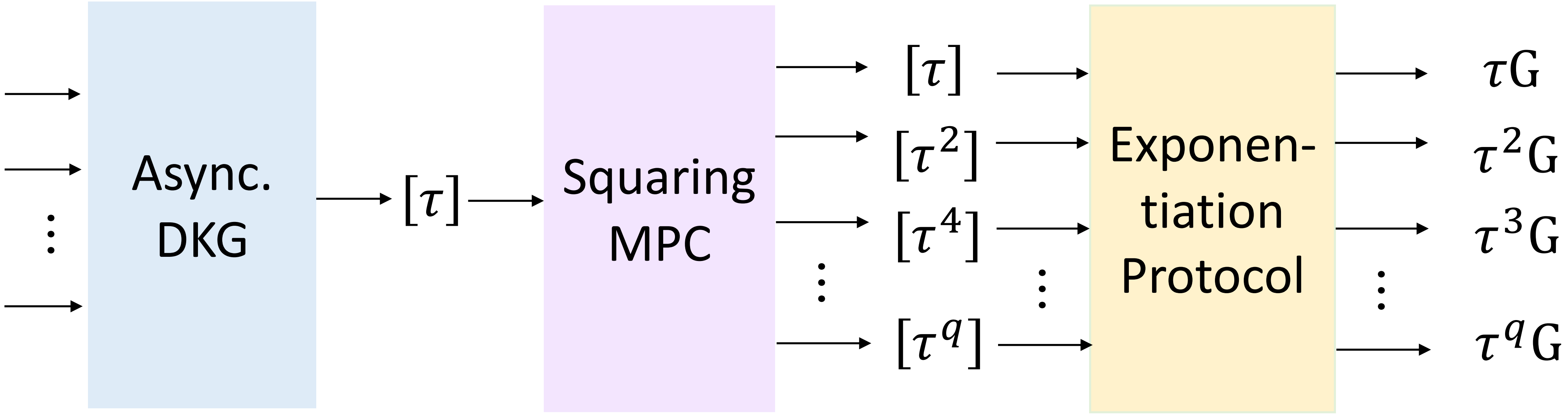

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All parts can be implemented with expected $\mathrm{O}(\log q + \log n)$ rounds

Step 1: Asynchronous DKG

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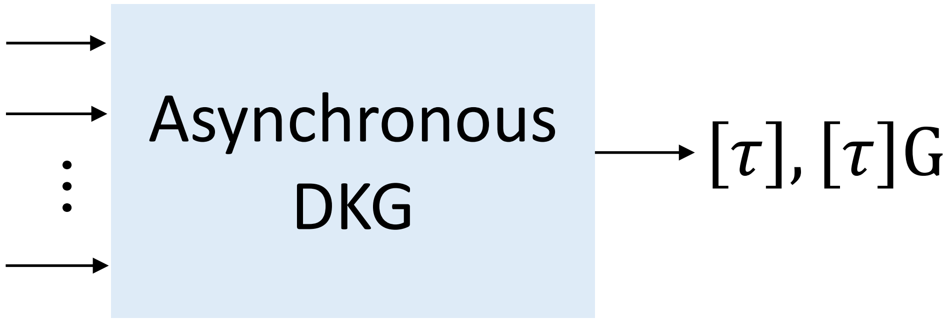

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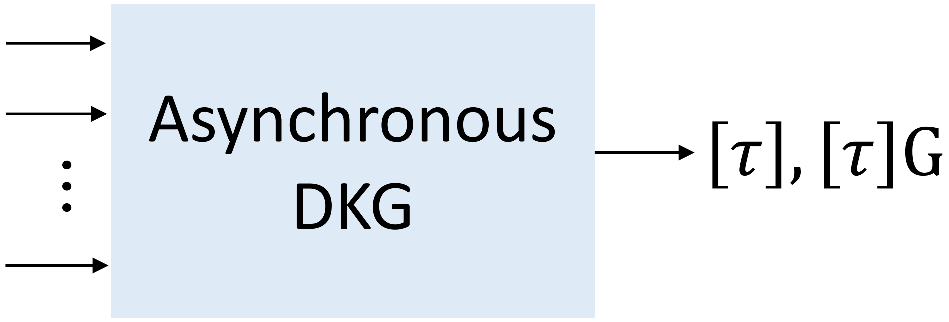

where $[\tau]\mathrm{G} = [\tau]_1\mathrm{G}, [\tau]_2\mathrm{G}, \ldots [\tau]_n\mathrm{G}$

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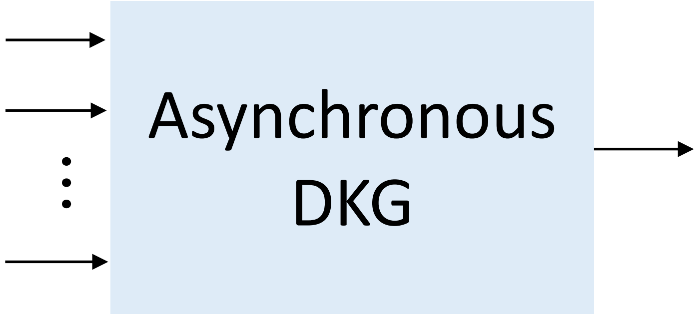

$[\tau], [\tau]\mathrm{G}$

where $[\tau]\mathrm{G} = [\tau]_1\mathrm{G}, [\tau]_2\mathrm{G}, \ldots [\tau]_n\mathrm{G}$

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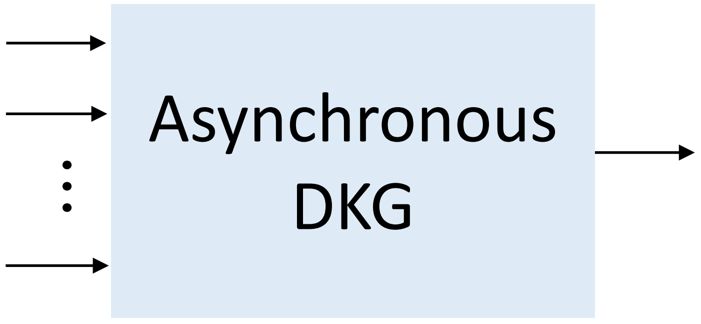

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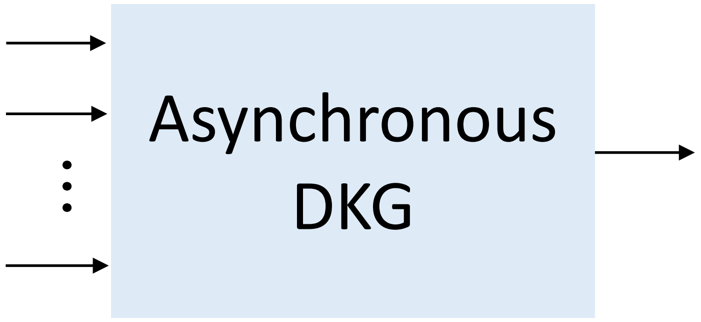

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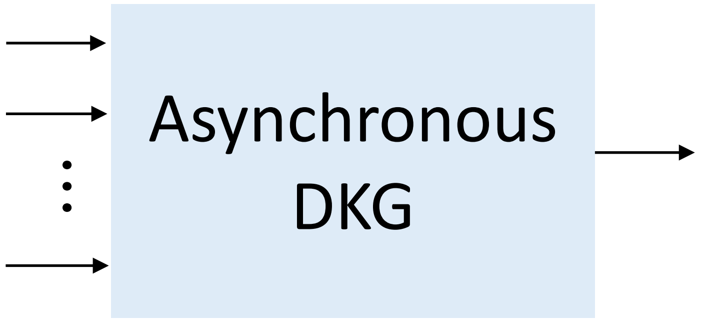

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Step 2: Squaring protocol

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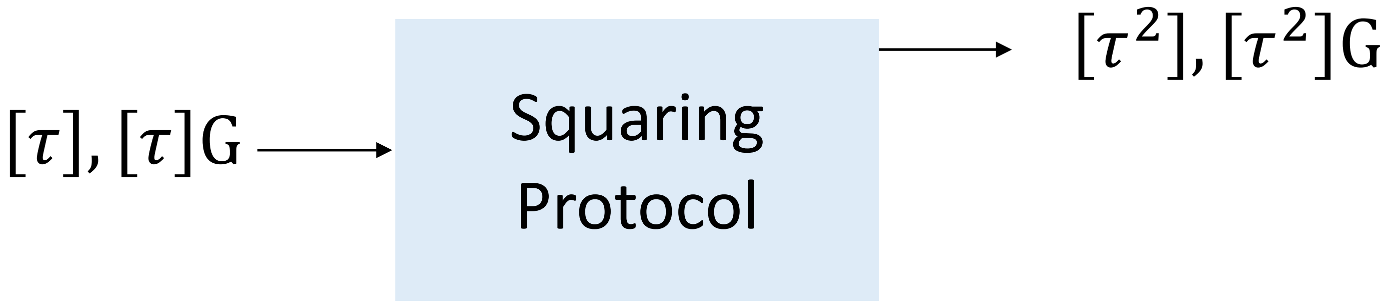
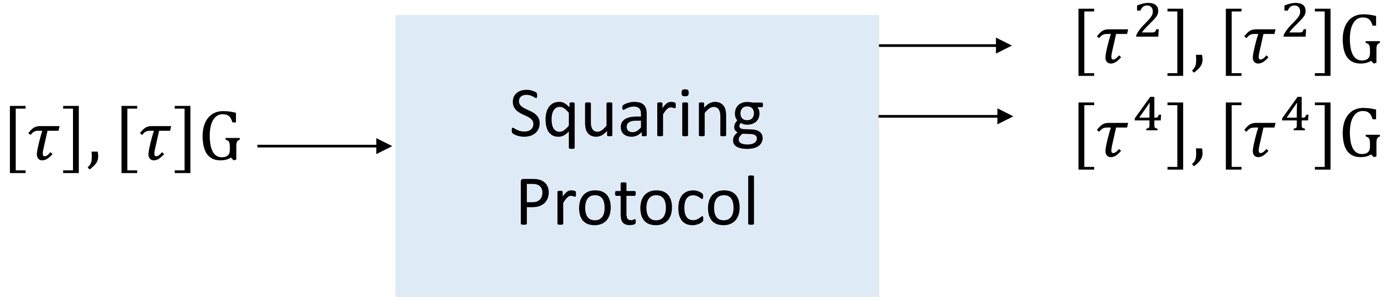
$[\tau], [\tau]\mathrm{G} \longrightarrow$ Squaring Protocol

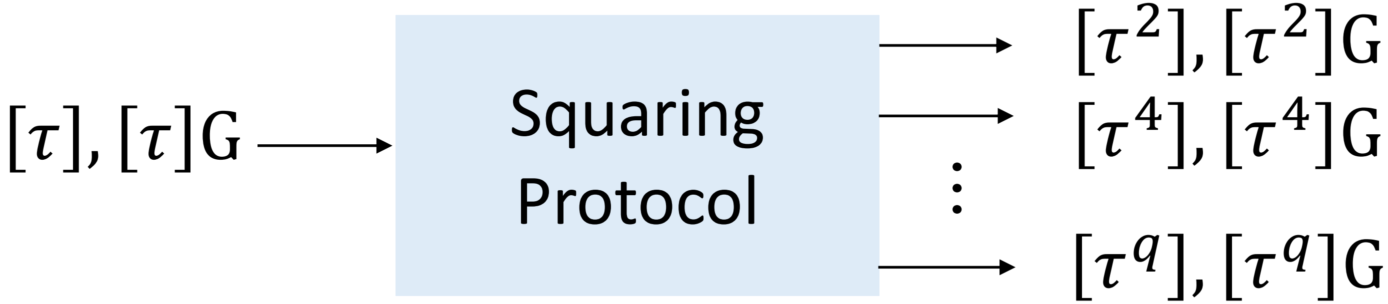
Step 2: Squaring protocol



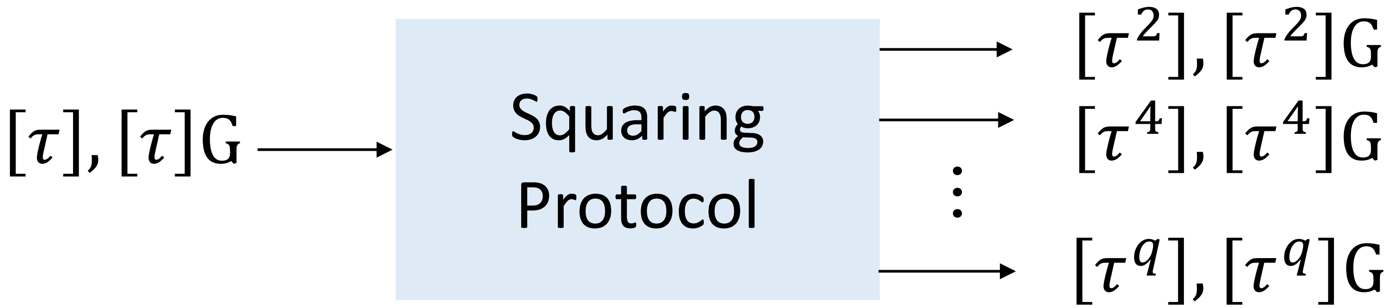
Step 2: Squaring protocol



Step 2: Squaring protocol

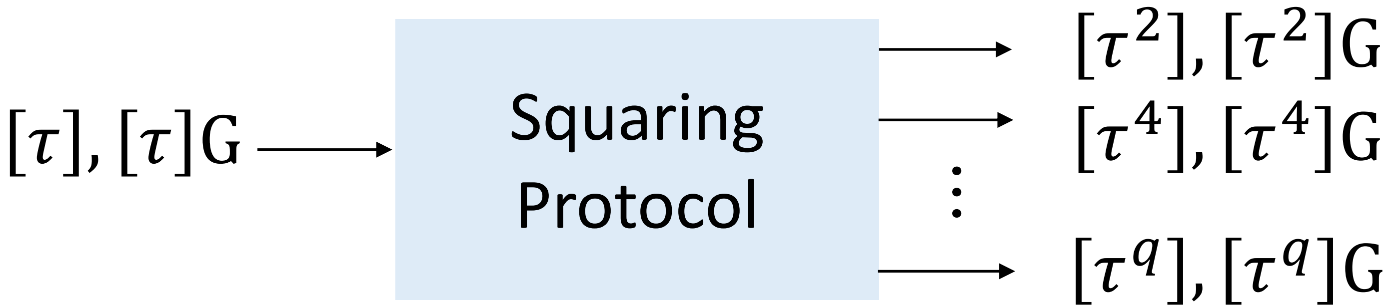


Step 2: Squaring protocol



Double sharing-based MPC multiplication

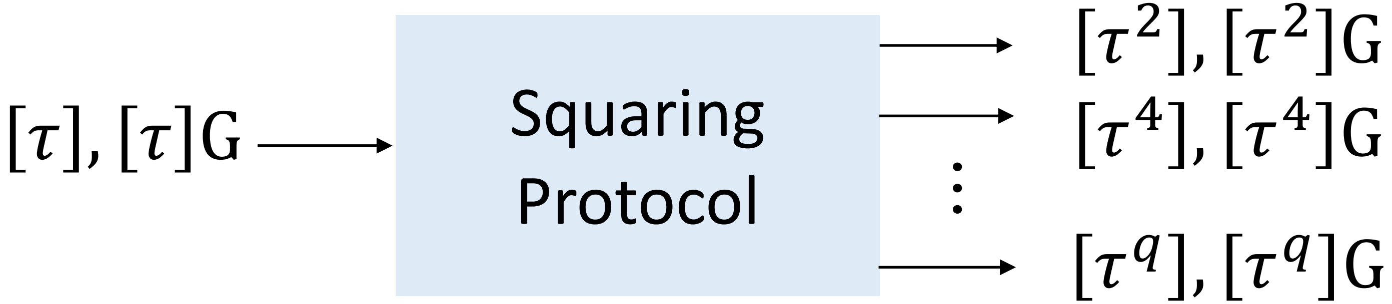
Step 2: Squaring protocol



Double sharing-based MPC multiplication

- Let $[a]^t$ and $[a]^{2t}$ be degree t and $2t$ sharing of a $a \leftarrow \mathbb{F}$

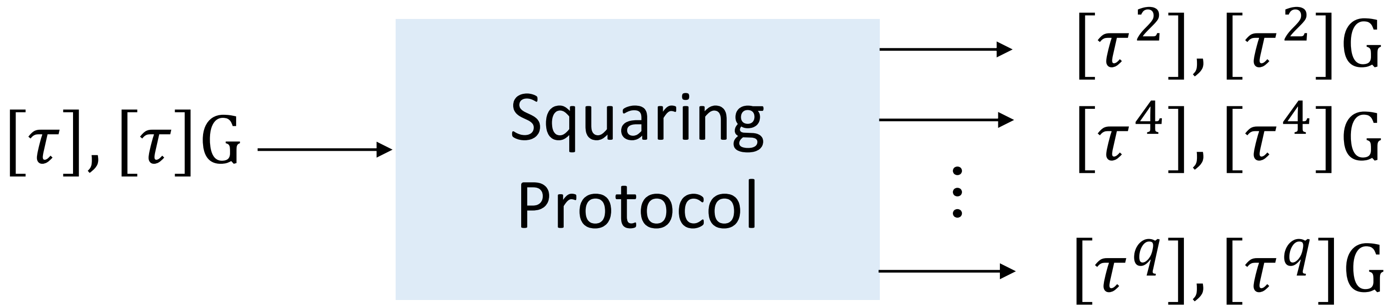
Step 2: Squaring protocol



Double sharing-based MPC multiplication

- Let $[a]^t$ and $[a]^{2t}$ be degree t and $2t$ sharing of a $a \leftarrow \mathbb{F}$
- Let $[a]^t\mathrm{G}$ and $[a]^{2t}\mathrm{G}$ be threshold public keys

Step 2: Squaring protocol



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$([\tau], [a]^{2t})$

Step 2: Squaring protocol

$[\tau], [\tau]G \longrightarrow$ **Squaring Protocol** $\longrightarrow [\tau^2], [\tau^2]G$; $[\tau^4], [\tau^4]G$; $\vdots$; $[\tau^q], [\tau^q]G$

Double sharing-based MPC multiplication

- Let $[a]^t$ and $[a]^{2t}$ be degree t and $2t$ sharing of a $a \leftarrow \mathbb{F}$
- Let $[a]^t\mathrm{G}$ and $[a]^{2t}\mathrm{G}$ be threshold public keys

$$([\tau], [a]^{2t}) \longrightarrow \boxed{\mathrm{Reveal}([\tau][\tau] + [a]^{2t})}$$

Step 2: Squaring protocol

$[\tau], [\tau]G \longrightarrow$ **Squaring Protocol** $\longrightarrow [\tau^2], [\tau^2]G$

$\longrightarrow [\tau^4], [\tau^4]G$

$\vdots$

$\longrightarrow [\tau^q], [\tau^q]G$

Double sharing-based MPC multiplication

- Let $[a]^t$ and $[a]^{2t}$ be degree t and $2t$ sharing of a $a \leftarrow \mathbb{F}$
- Let $[a]^t G$ and $[a]^{2t} G$ be threshold public keys

$$([\tau], [a]^{2t}) \longrightarrow \boxed{\text{Reveal}([\tau][\tau] + [a]^{2t})} \longrightarrow z$$

Step 2: Squaring protocol

$[\tau], [\tau]\mathrm{G} \longrightarrow$ **Squaring Protocol** $\longrightarrow$

$[\tau^2], [\tau^2]\mathrm{G}$

$[\tau^4], [\tau^4]\mathrm{G}$

$\vdots$

$[\tau^q], [\tau^q]\mathrm{G}$

Double sharing-based MPC multiplication

- Let $[a]^t$ and $[a]^{2t}$ be degree t and $2t$ sharing of a $a \leftarrow \mathbb{F}$
- Let $[a]^t\mathrm{G}$ and $[a]^{2t}\mathrm{G}$ be threshold public keys

$$([\tau], [a]^{2t}) \longrightarrow \mathrm{Reveal}([\tau][\tau] + [a]^{2t}) \longrightarrow z$$

Step 2: Squaring protocol

$$[\tau], [\tau]\mathrm{G} \longrightarrow \boxed{\text{Squaring Protocol}} \longrightarrow \begin{matrix} [\tau^2], [\tau^2]\mathrm{G} \\ [\tau^4], [\tau^4]\mathrm{G} \\ \vdots \\ [\tau^q], [\tau^q]\mathrm{G} \end{matrix}$$

Double sharing-based MPC multiplication

- Let $[a]^t$ and $[a]^{2t}$ be degree t and $2t$ sharing of a $a \leftarrow \mathbb{F}$
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$$([\tau], [a]^{2t}) \longrightarrow \boxed{\mathrm{Reveal}([\tau][\tau] + [a]^{2t})} \longrightarrow z$$

Compute $[\tau^2]_i \coloneqq z - [a]_i^t$

Step 2: Squaring protocol

$[\tau], [\tau]\mathrm{G} \longrightarrow$ Squaring Protocol $\longrightarrow [\tau^2], [\tau^2]\mathrm{G}$; $[\tau^4], [\tau^4]\mathrm{G}$; $\vdots$; $[\tau^q], [\tau^q]\mathrm{G}$

Double sharing-based MPC multiplication

- Let $[a]^t$ and $[a]^{2t}$ be degree t and $2t$ sharing of a $a \leftarrow \mathbb{F}$
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$$([\tau], [a]^{2t}) \longrightarrow \mathrm{Reveal}([\tau][\tau] + [a]^{2t}) \longrightarrow z$$

Compute $[\tau^2]_i \coloneqq z - [a]_i^t$
Compute $[\tau^2]\mathrm{G} \coloneqq (z - [a]^t) \cdot \mathrm{G}$

Step 2: Squaring protocol

$$[\tau], [\tau]\mathrm{G} \longrightarrow \boxed{\text{Squaring Protocol}} \longrightarrow \begin{array}{l} [\tau^2], [\tau^2]\mathrm{G} \\ [\tau^4], [\tau^4]\mathrm{G} \\ \vdots \\ [\tau^q], [\tau^q]\mathrm{G} \end{array}$$

Double sharing-based MPC multiplication

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Double sharing generation from [DXKR'23]

Step 2: Squaring protocol

$[\tau], [\tau]\mathrm{G} \longrightarrow$ Squaring Protocol $\longrightarrow [\tau^2], [\tau^2]\mathrm{G}$

$\longrightarrow [\tau^4], [\tau^4]\mathrm{G}$

$\vdots$

$\longrightarrow [\tau^q], [\tau^q]\mathrm{G}$

Double sharing-based MPC multiplication

- Let $[a]^t$ and $[a]^{2t}$ be degree t and $2t$ sharing of a $a \leftarrow \mathbb{F}$
- Let $[a]^t\mathrm{G}$ and $[a]^{2t}\mathrm{G}$ be threshold public keys

$$([\tau], [a]^{2t}) \longrightarrow \mathrm{Reveal}([\tau][\tau] + [a]^{2t}) \longrightarrow z$$

Compute $[\tau^2]_i := z - [a]_i^t$
Compute $[\tau^2]\mathrm{G} := (z - [a]^t) \cdot \mathrm{G}$

Double sharing generation from [DXKR'23]

- Per party per unit communication cost of $O(n^2)$

Step 2: Squaring protocol

$[\tau], [\tau]\mathrm{G} \longrightarrow$ **Squaring Protocol** $\longrightarrow$

$[\tau^2], [\tau^2]\mathrm{G}$

$[\tau^4], [\tau^4]\mathrm{G}$

$\vdots$

$[\tau^q], [\tau^q]\mathrm{G}$

Double sharing-based MPC multiplication

- Let $[a]^t$ and $[a]^{2t}$ be degree t and $2t$ sharing of a $a \leftarrow \mathbb{F}$
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$([\tau], [a]^{2t}) \longrightarrow \mathrm{Reveal}([\tau][\tau] + [a]^{2t}) \longrightarrow z$

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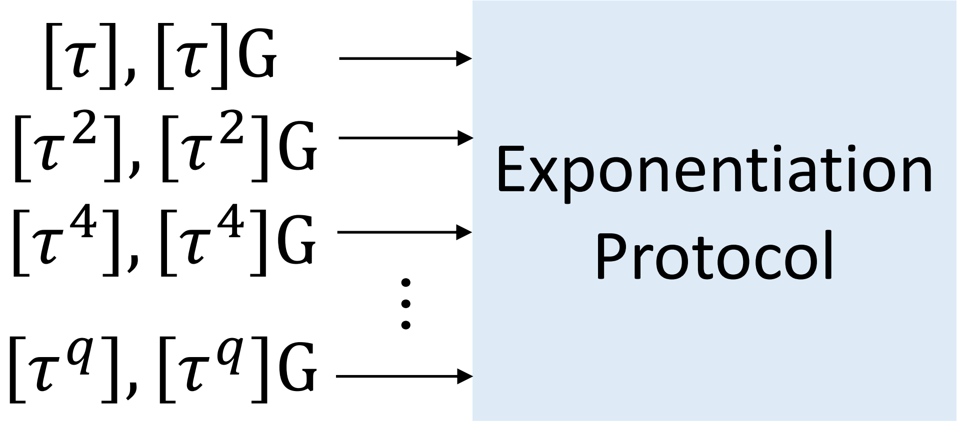
Double sharing generation from [DXKR'23]

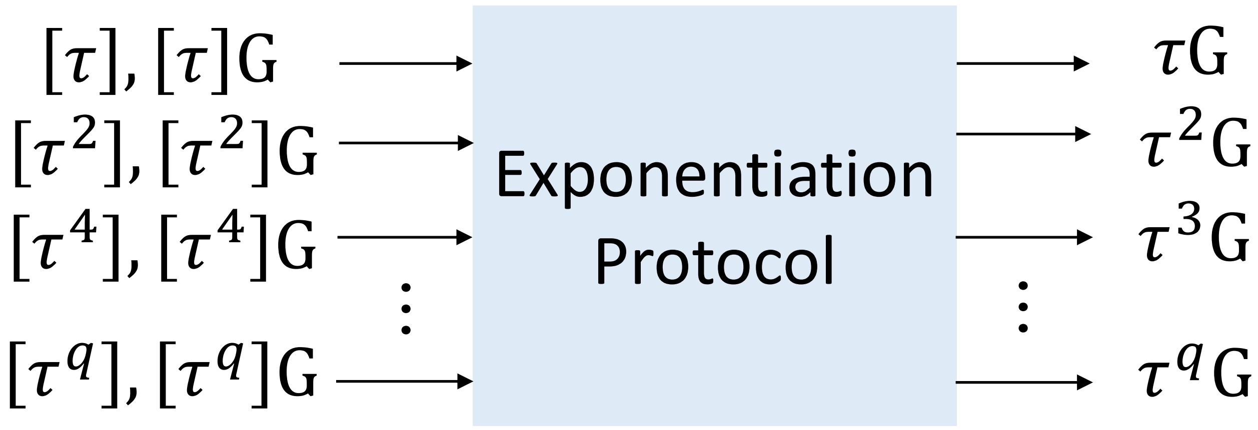
- Per party per unit communication cost of $O(n^2)$
- Per party total communication cost of $O(n^2 \log q)$

Step 3: All exponents

Step 3: All exponents



Step 3: All exponents



Step 3: All exponents

$[\tau], [\tau]G \longrightarrow$ Exponentiation Protocol $\longrightarrow \tau G$

$[\tau^2], [\tau^2]G \longrightarrow$ $\longrightarrow \tau^2 G$

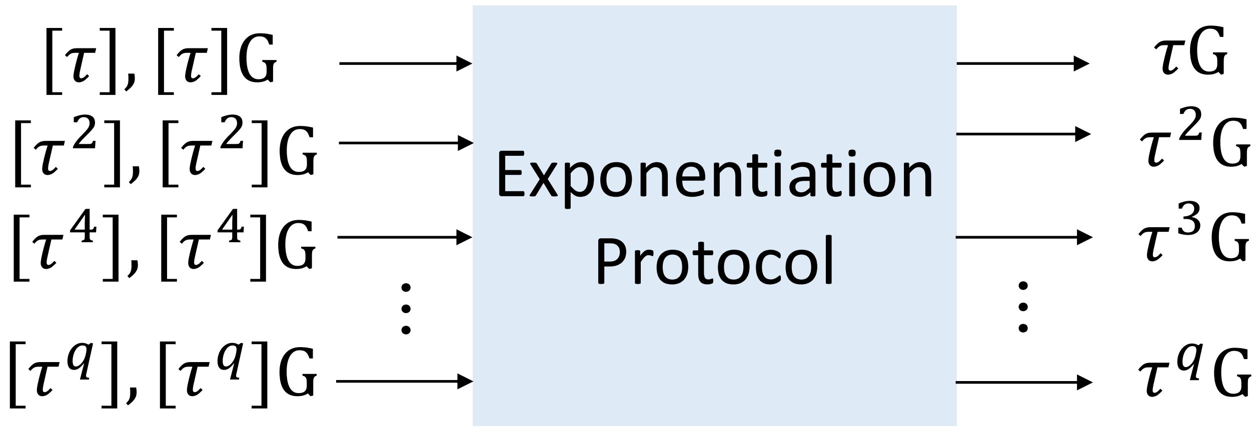
$[\tau^4], [\tau^4]G \longrightarrow$ $\longrightarrow \tau^3 G$

$\vdots$ $\vdots$

$[\tau^q], [\tau^q]G \longrightarrow$ $\longrightarrow \tau^q G$

Example: $\tau^5 G$

Step 3: All exponents



Example: $\tau^5\mathrm{G} = (\tau^{2\cdot 2}\tau)\mathrm{G}$

Step 3: All exponents

Input		Output
$[\tau], [\tau]\mathrm{G}$	Exponentiation Protocol	$\tau\mathrm{G}$
$[\tau^2], [\tau^2]\mathrm{G}$		$\tau^2\mathrm{G}$
$[\tau^4], [\tau^4]\mathrm{G}$		$\tau^3\mathrm{G}$
$\vdots$		$\vdots$
$[\tau^q], [\tau^q]\mathrm{G}$		$\tau^q\mathrm{G}$

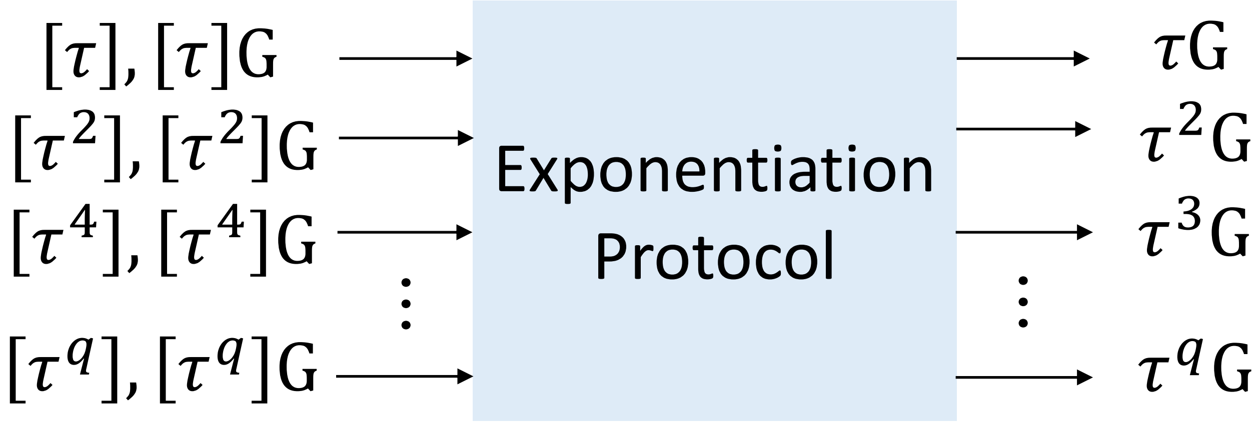
Example: $\tau^5\mathrm{G} = (\tau^{2\cdot 2}\tau)\mathrm{G} = \tau^2(\tau^2\mathrm{G}) \cdot (\tau\mathrm{G})$

Step 3: All exponents

$$
\begin{array}{ccccc}
[\tau], [\tau]\mathrm{G} & \longrightarrow & & \longrightarrow & \tau\mathrm{G} \\
[\tau^2], [\tau^2]\mathrm{G} & \longrightarrow & \text{Exponentiation} & \longrightarrow & \tau^2\mathrm{G} \\
[\tau^4], [\tau^4]\mathrm{G} & \longrightarrow & \text{Protocol} & \longrightarrow & \tau^3\mathrm{G} \\
 & \vdots & & \vdots & \\
[\tau^q], [\tau^q]\mathrm{G} & \longrightarrow & & \longrightarrow & \tau^q\mathrm{G}
\end{array}
$$

Example: $\tau^5\mathrm{G} = (\tau^{2\cdot 2}\tau)\mathrm{G} = \tau^2(\tau^2\mathrm{G}) \cdot (\tau\mathrm{G})$

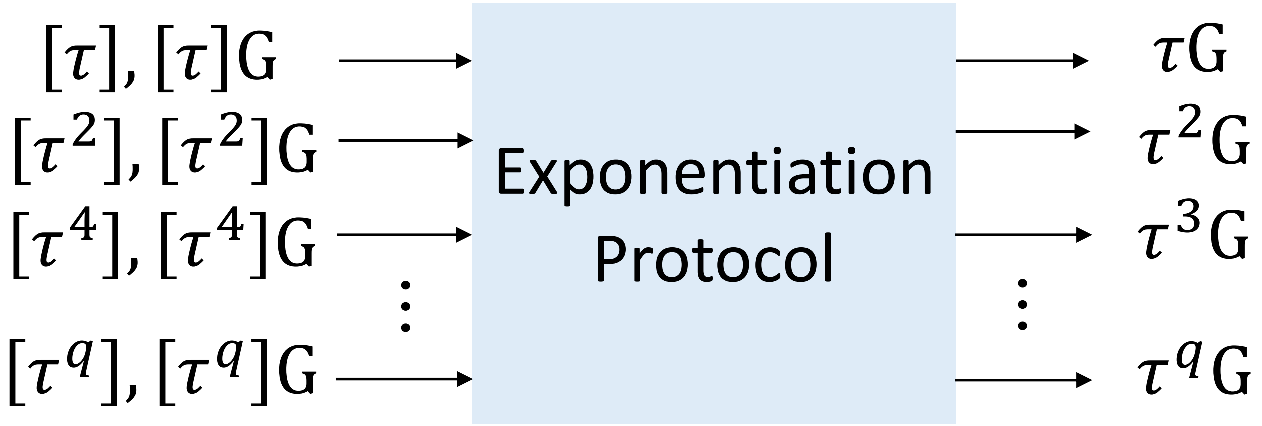
Step 3: All exponents



Example: $\tau^5\mathrm{G} = (\tau^{2\cdot 2}\tau)\mathrm{G} \ = \tau^2(\tau^2\mathrm{G}) \cdot (\tau\mathrm{G})$

Protocol:

Step 3: All exponents



Example: $\tau^5 G = (\tau^{2 \cdot 2}\tau)G \; = \tau^2(\tau^2 G) \cdot (\tau G)$

Protocol:

1. Each node i publishes $[\tau^2]_i(\tau^2 G)$

Step 3: All exponents

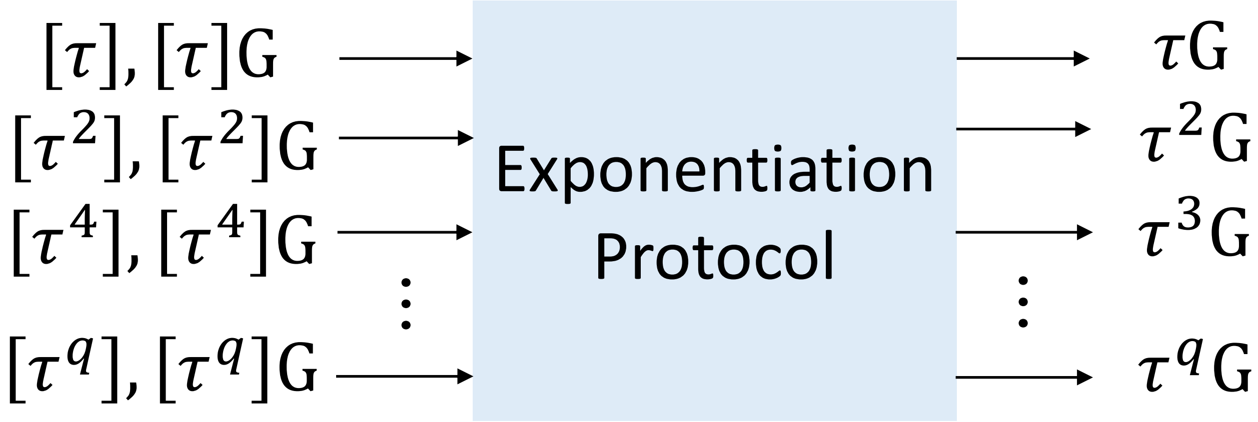
$$\begin{array}{rcl}
[\tau], [\tau]\mathrm{G} \longrightarrow & & \longrightarrow \tau\mathrm{G} \\
[\tau^2], [\tau^2]\mathrm{G} \longrightarrow & \text{Exponentiation} & \longrightarrow \tau^2\mathrm{G} \\
[\tau^4], [\tau^4]\mathrm{G} \longrightarrow & \text{Protocol} & \longrightarrow \tau^3\mathrm{G} \\
\vdots & & \vdots \\
[\tau^q], [\tau^q]\mathrm{G} \longrightarrow & & \longrightarrow \tau^q\mathrm{G}
\end{array}$$

Example: $\tau^5\mathrm{G} = (\tau^{2\cdot 2}\tau)\mathrm{G} \ = \tau^2(\tau^2\mathrm{G}) \cdot (\tau\mathrm{G})$

Protocol:

1. Each node i publishes $[\tau^2]_i(\tau^2\mathrm{G})$
2. Interpolate $[\tau^2]_i(\tau^2\mathrm{G})$ in the exponent to compute $\tau^2(\tau^2\mathrm{G})$

Step 3: All exponents



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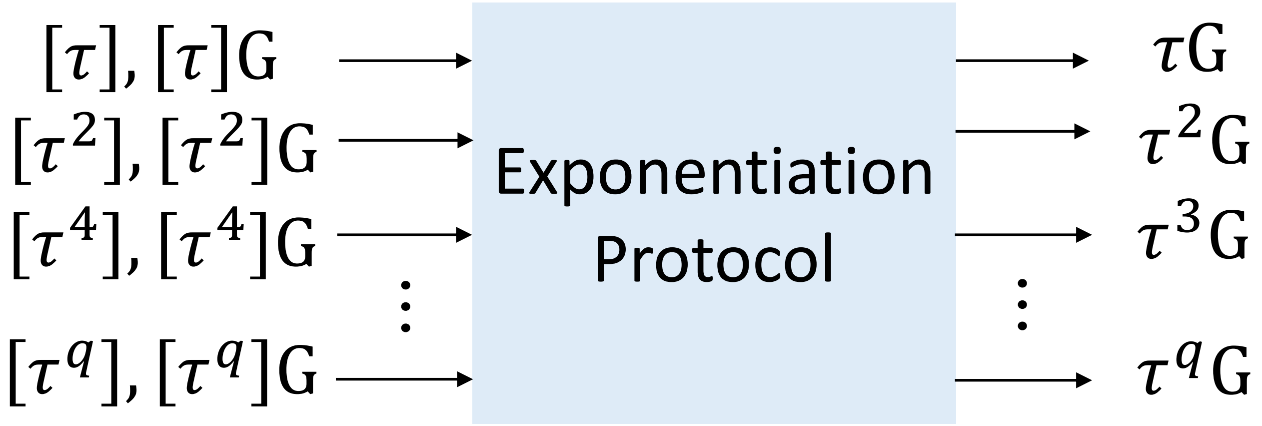
3. Compute $\tau^2(\tau^2\mathrm{G}) \cdot (\tau\mathrm{G}) = \tau^5\mathrm{G}$

Step 3: All exponents



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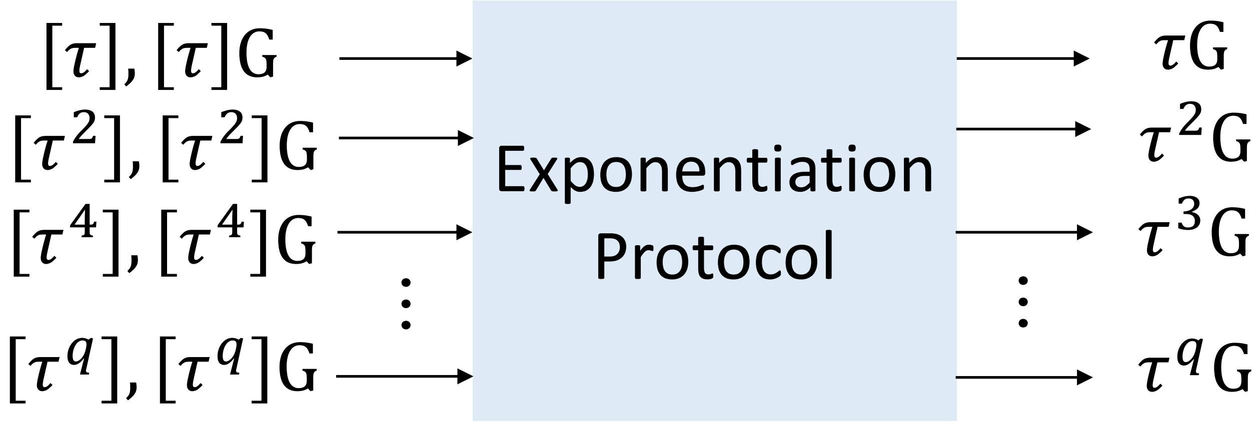

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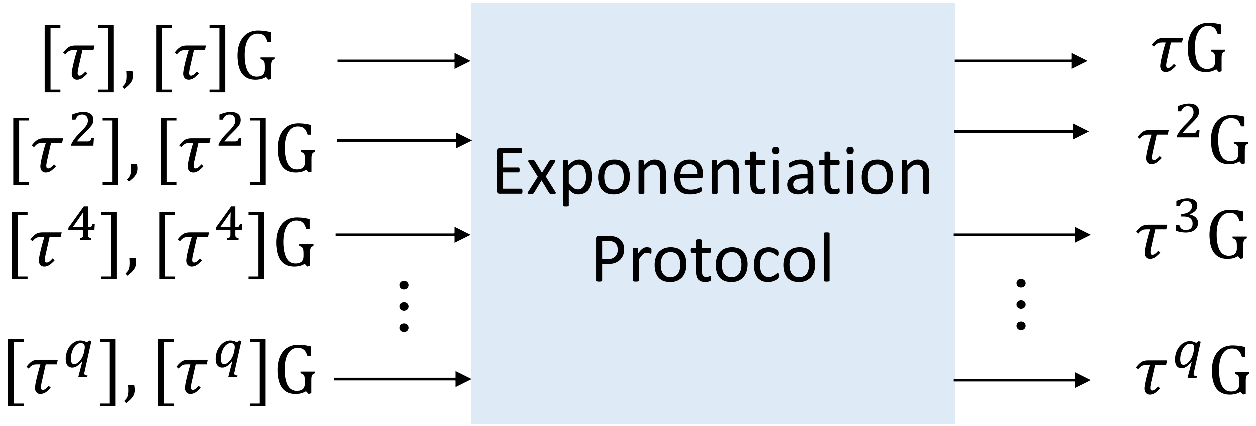
- Naively $\mathrm{O}(n)$ per-party communication per exponent

Step 3: All exponents



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- Naively $\mathrm{O}(n)$ per-party communication per exponent
- Batch amortization optimization to get $\mathrm{O}(1)$ per-party communication cost

Putting together everything

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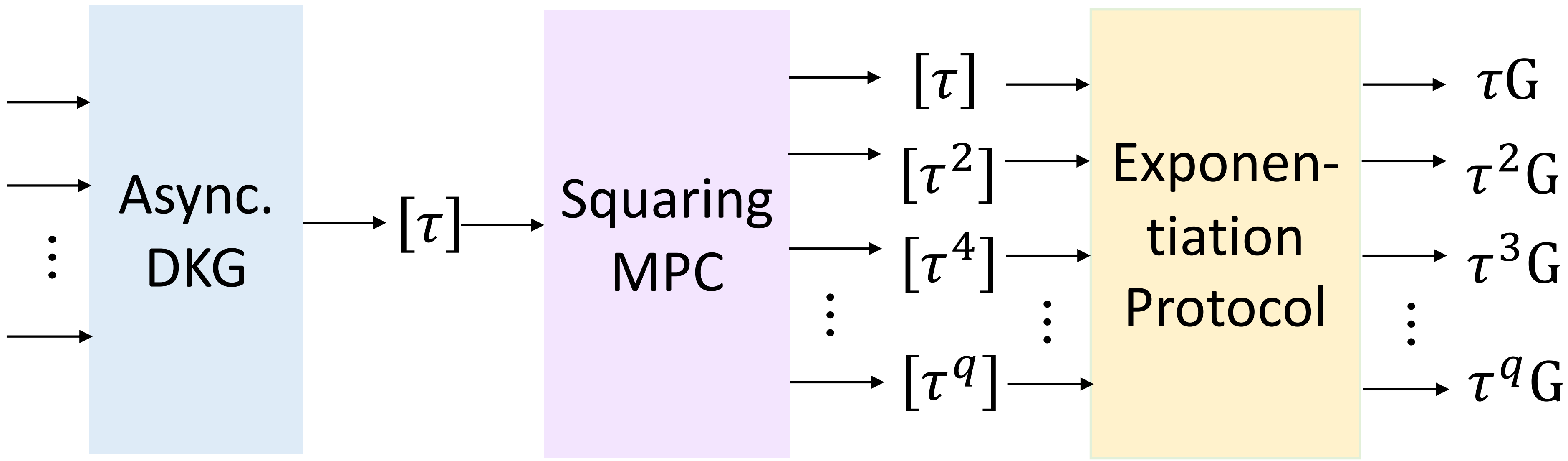

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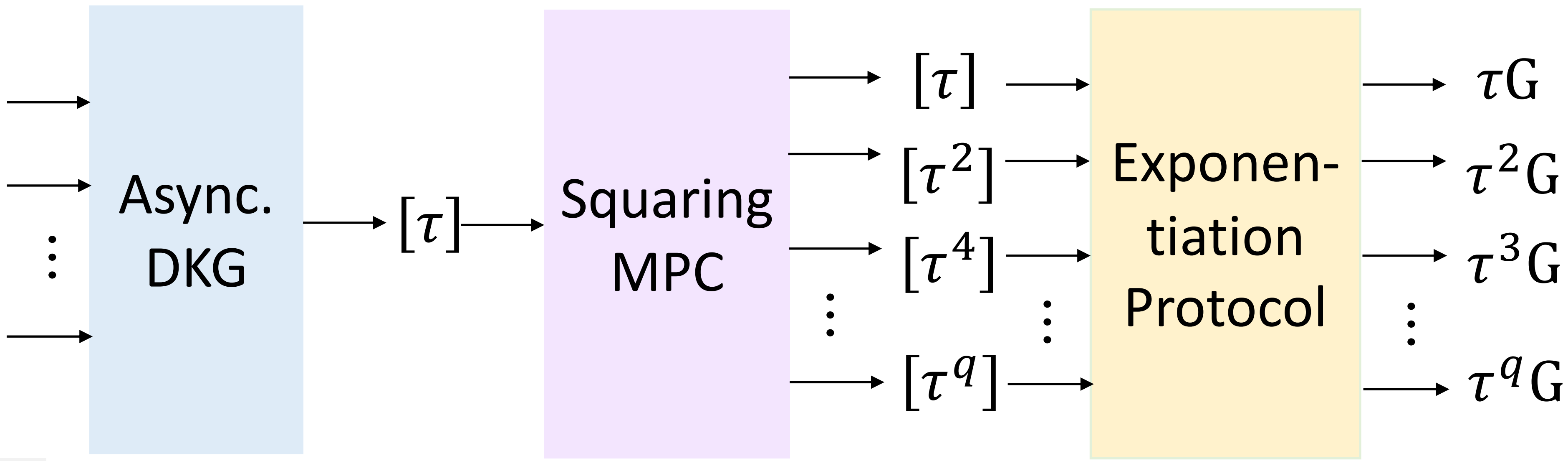
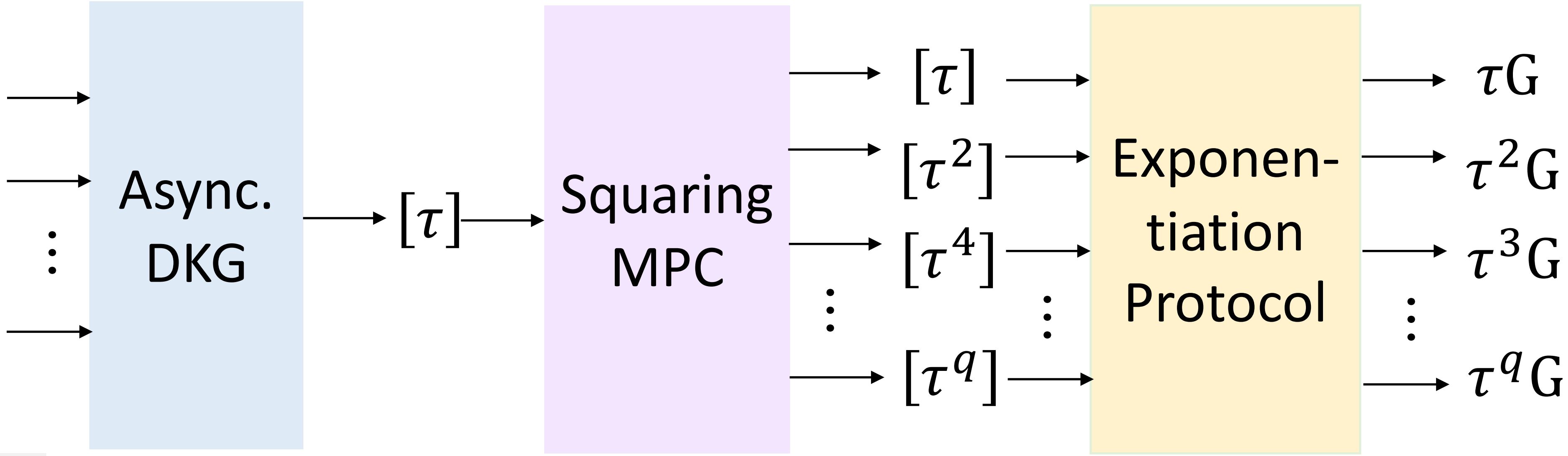


Comm. Cost:

Rounds:

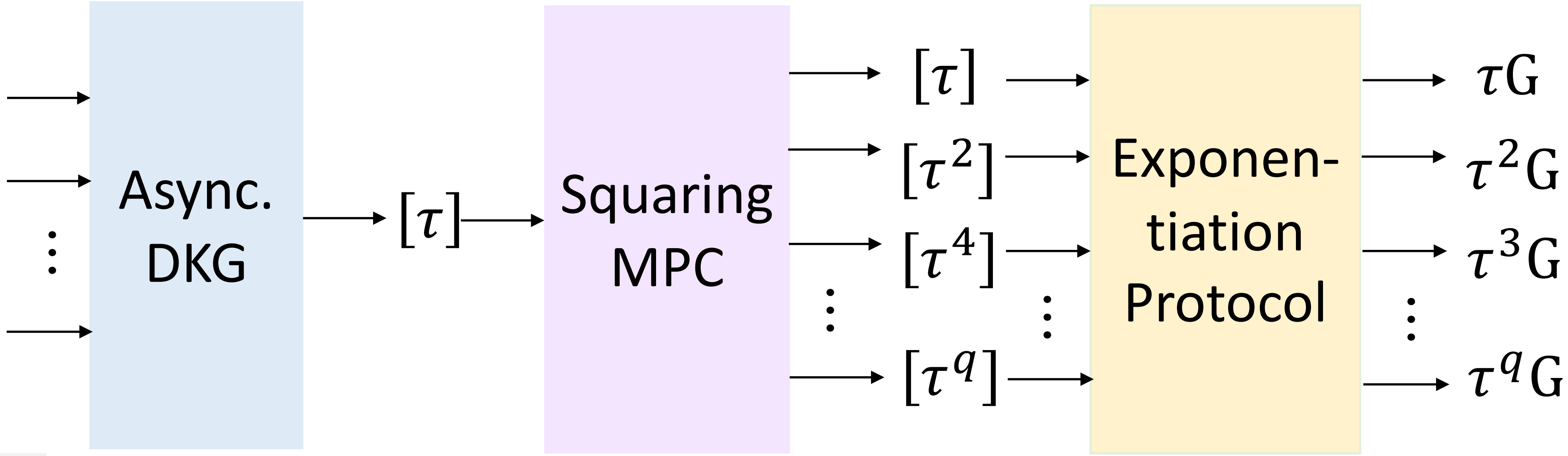
Putting together everything



Comm. Cost: $O(n^2)$

Rounds:

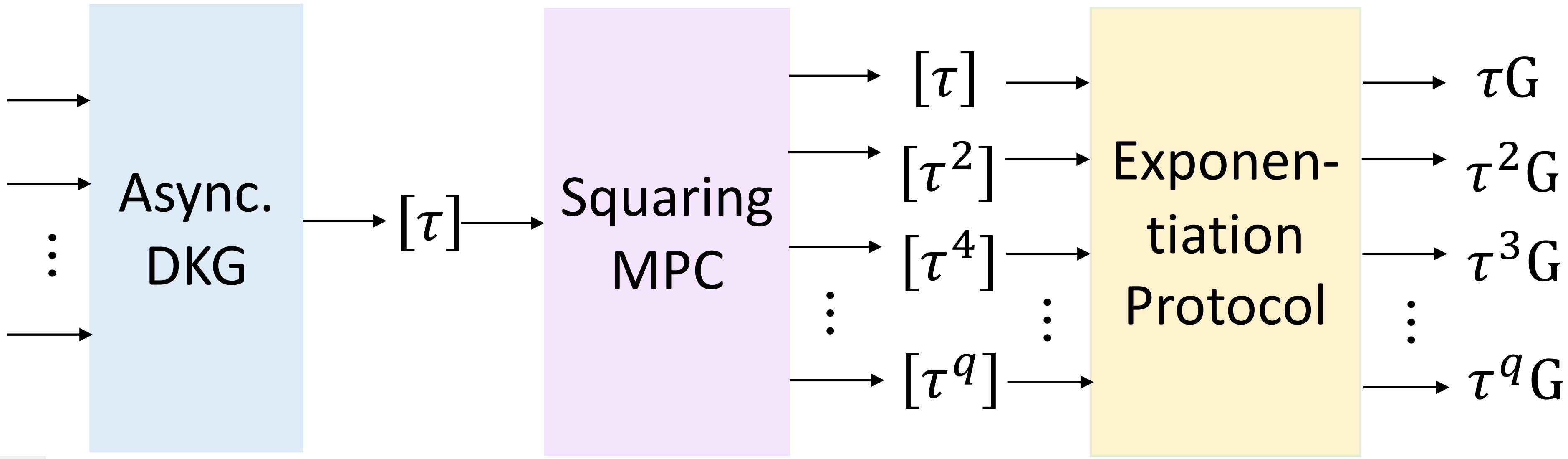
Putting together everything



Comm. Cost: $O(n^2)$

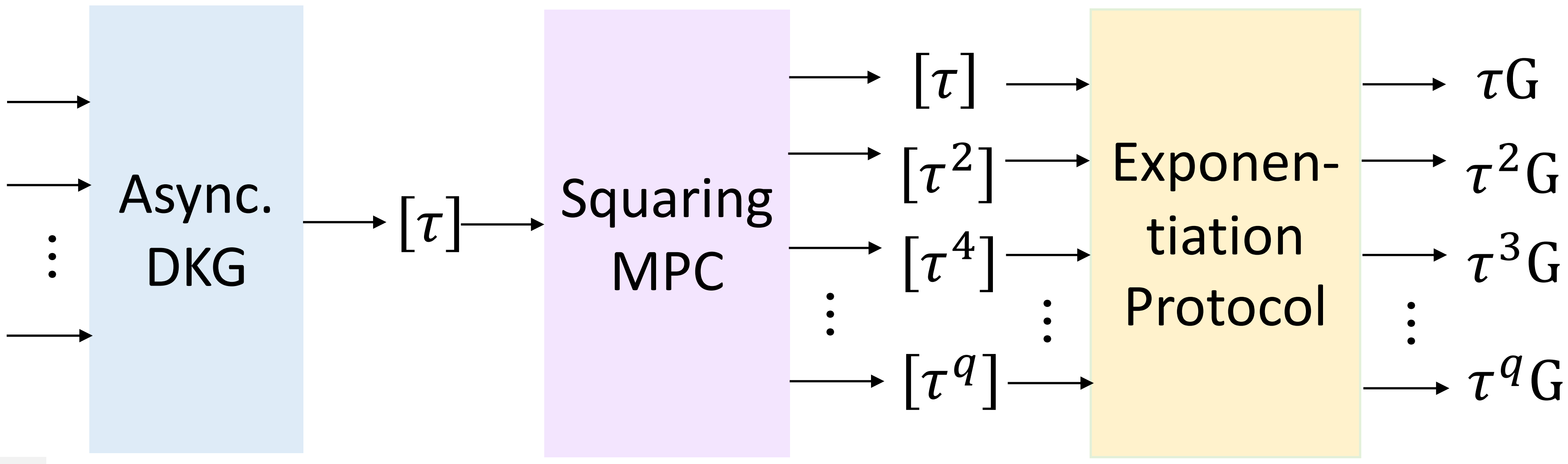
Rounds: $O(\log n)$

Putting together everything



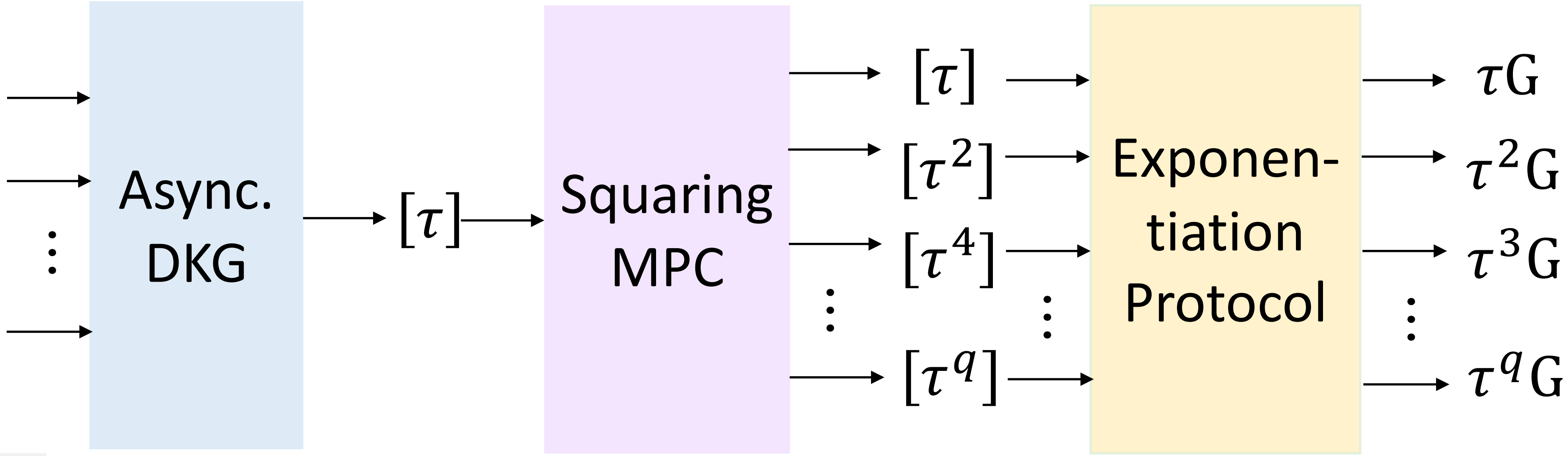
Comm. Cost:	$O(n^2)$	$O(n^2 \log q)$
Rounds:	$O(\log n)$	

Putting together everything



	Async. DKG	Squaring MPC
Comm. Cost:	$O(n^2)$	$O(n^2 \log q)$
Rounds:	$O(\log n)$	$O(\log q)$

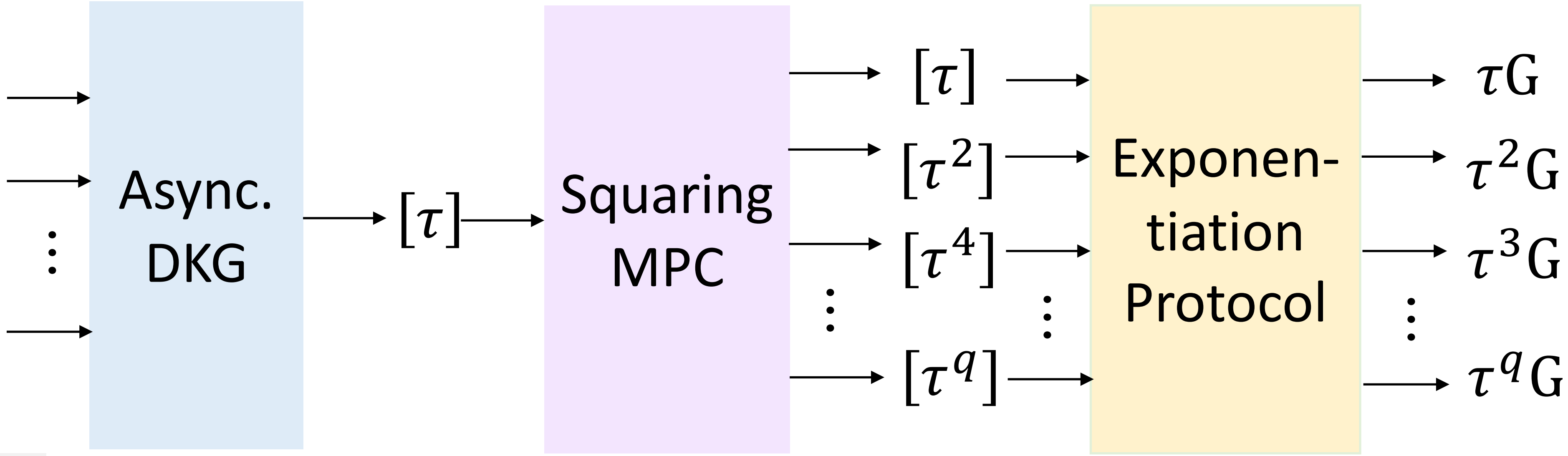
Putting together everything



Comm. Cost: $O(n^2)$ $O(n^2 \log q)$ $O(q)$

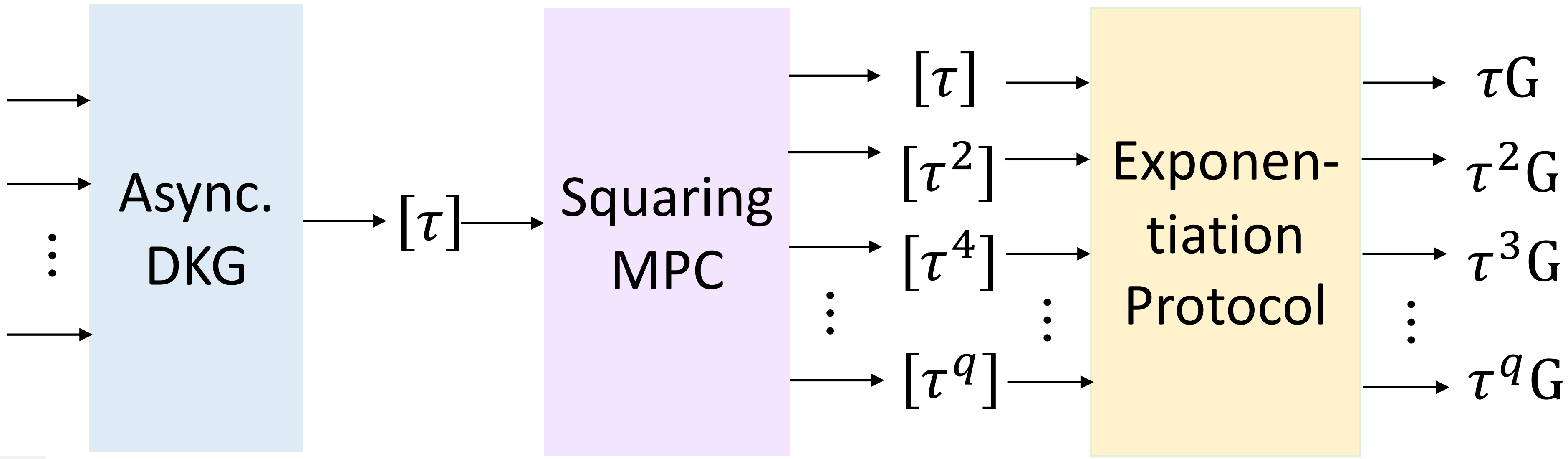
Rounds: $O(\log n)$ $O(\log q)$

Putting together everything



	Async. DKG	Squaring MPC	Exponentiation Protocol
Comm. Cost:	$O(n^2)$	$O(n^2 \log q)$	$O(q)$
Rounds:	$O(\log n)$	$O(\log q)$	$O(\log q)$

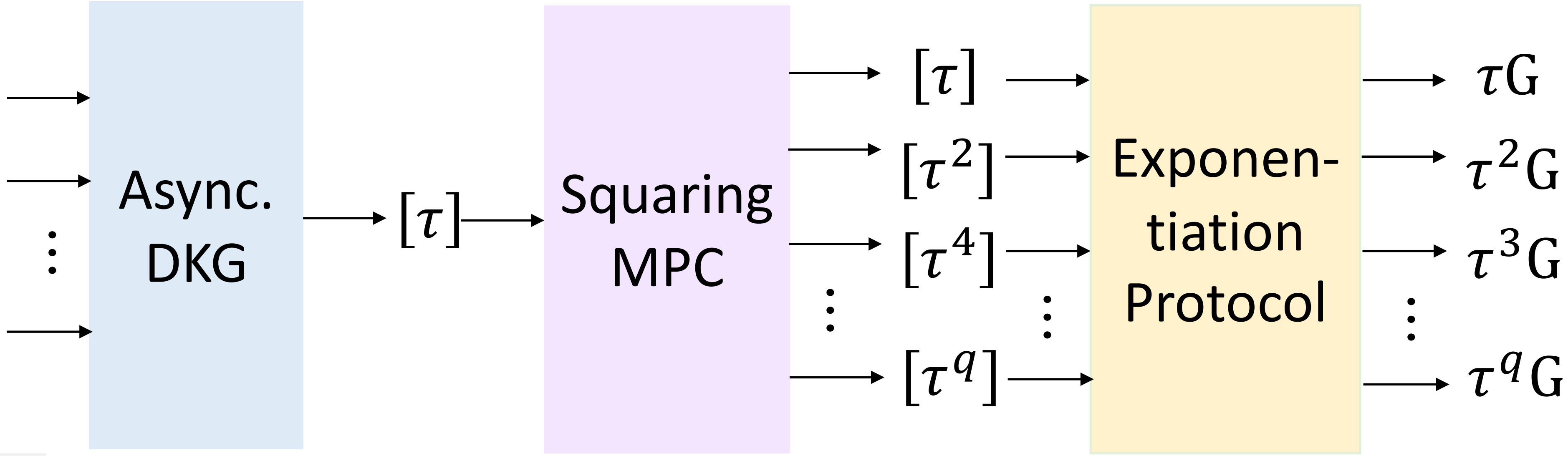
Putting together everything



Comm. Cost:	$O(n^2)$	$O(n^2 \log q)$	$O(q)$
Rounds:	$O(\log n)$	$O(\log q)$	$O(\log q)$

Total per party communication cost: $O(q + n^2 \log q)$

Putting together everything



	Async. DKG	Squaring MPC	Exponentiation Protocol
Comm. Cost:	$O(n^2)$	$O(n^2 \log q)$	$O(q)$
Rounds:	$O(\log n)$	$O(\log q)$	$O(\log q)$

Total per party communication cost: $O(q + n^2 \log q)$

Expected rounds: $O(\log n + \log q)$. Can me made $O(\log q)$

Implementation and Evaluation

Implementation Details

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- Implemented in python with rust for cryptography

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Implementation Details

- Implemented in python with rust for cryptography
- Available at https://github.com/sourav1547/qsdh-py
- Evaluation with up to 128 AWS nodes
- Round-robin protocol as baseline
  - $n|M|$ as bandwidth usage of broadcast
  - Computation cost of broadcast is free

Evaluation results: Runtime

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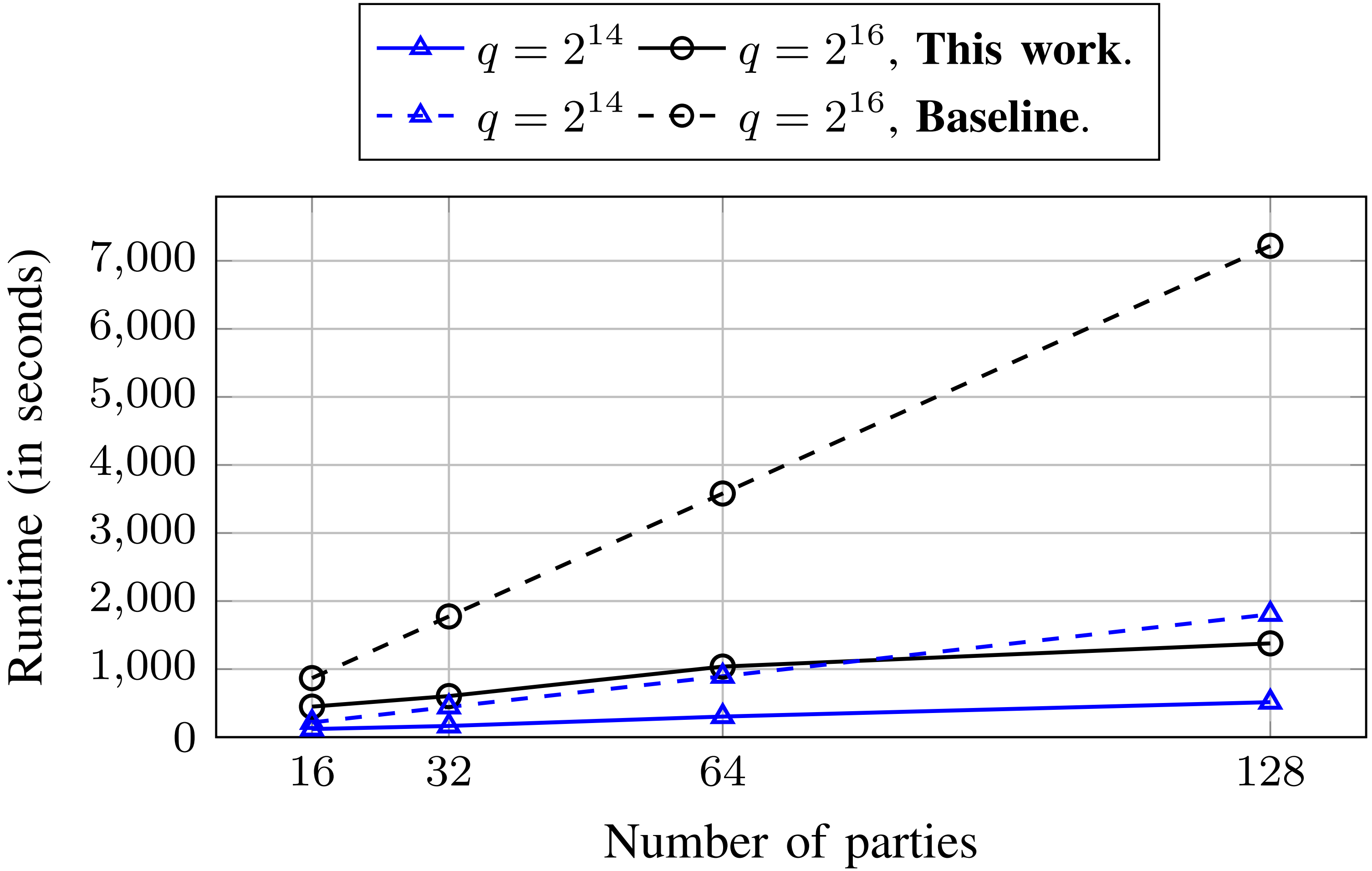
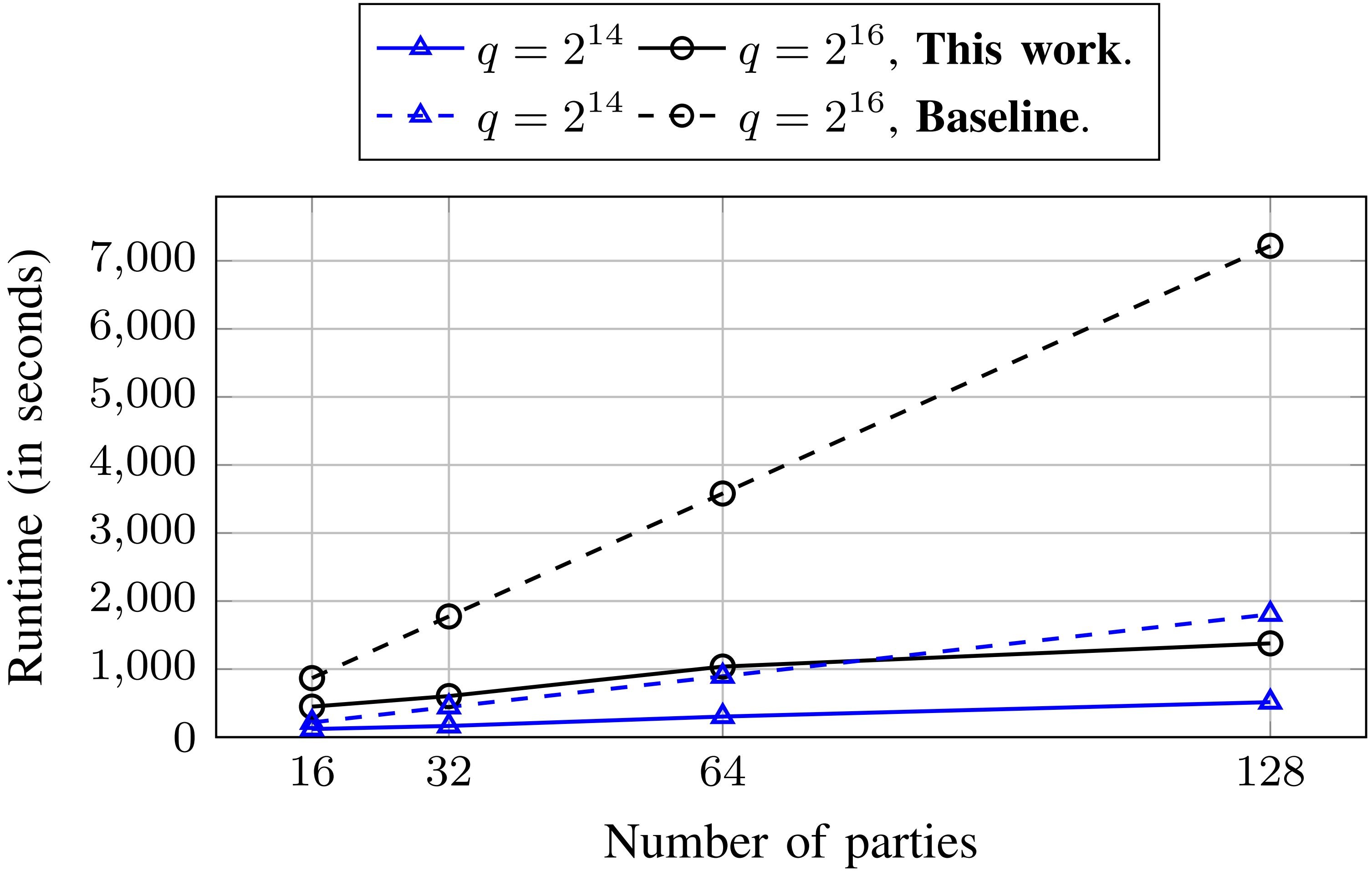
$q = 2^{14}$ $q = 2^{16}$, **This work**.

$q = 2^{14}$ $q = 2^{16}$, **Baseline**.

Evaluation results: Runtime



Evaluation results: Runtime



For example, with $q = 2^{16}$, Ours: 1037 seconds, Baseline: 3580 seconds (3.4×)

Evaluation results: Bandwidth Usage

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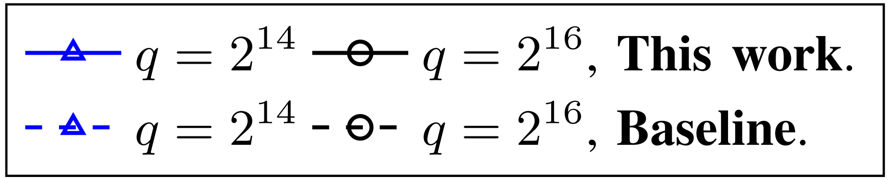

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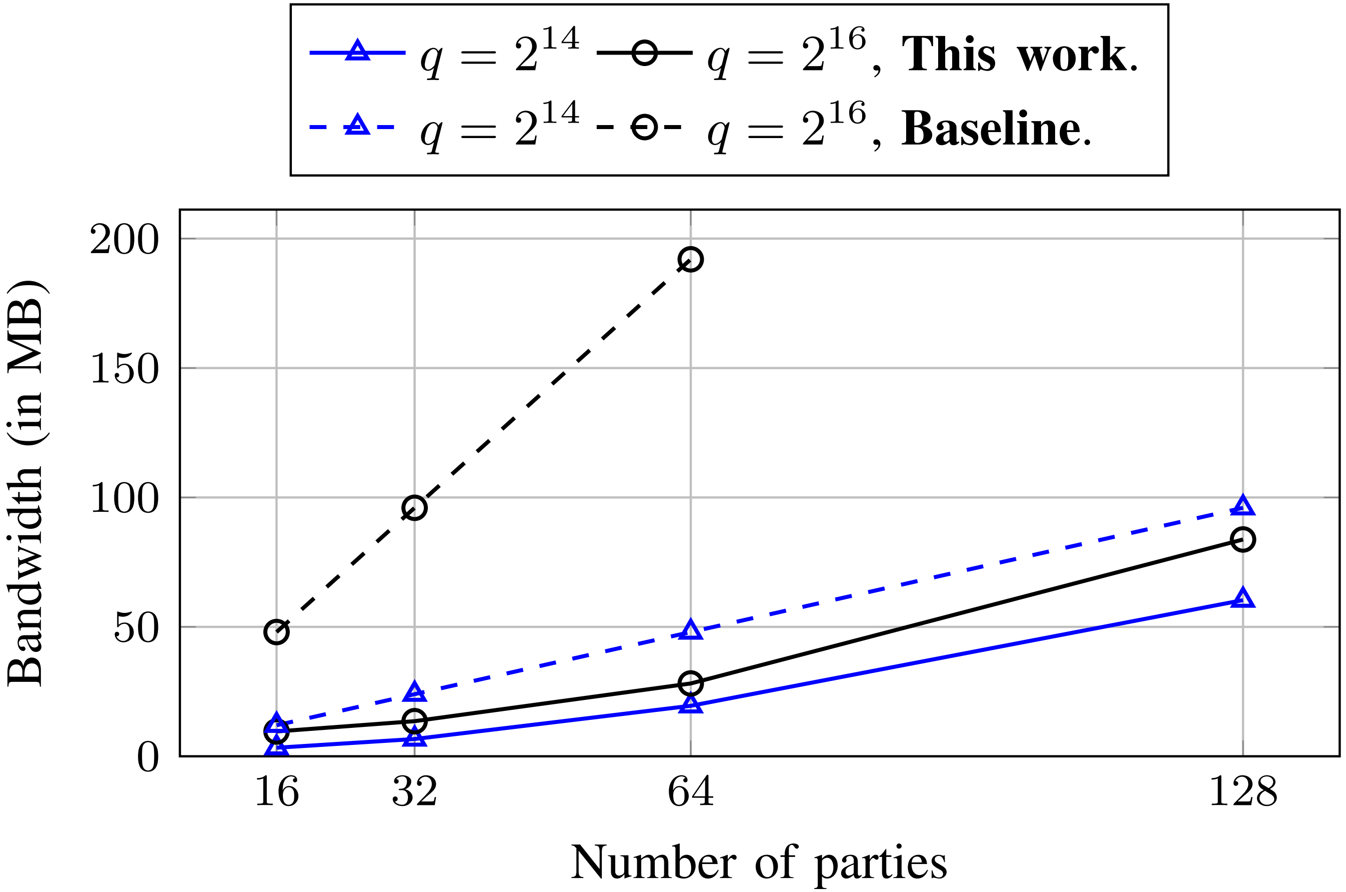

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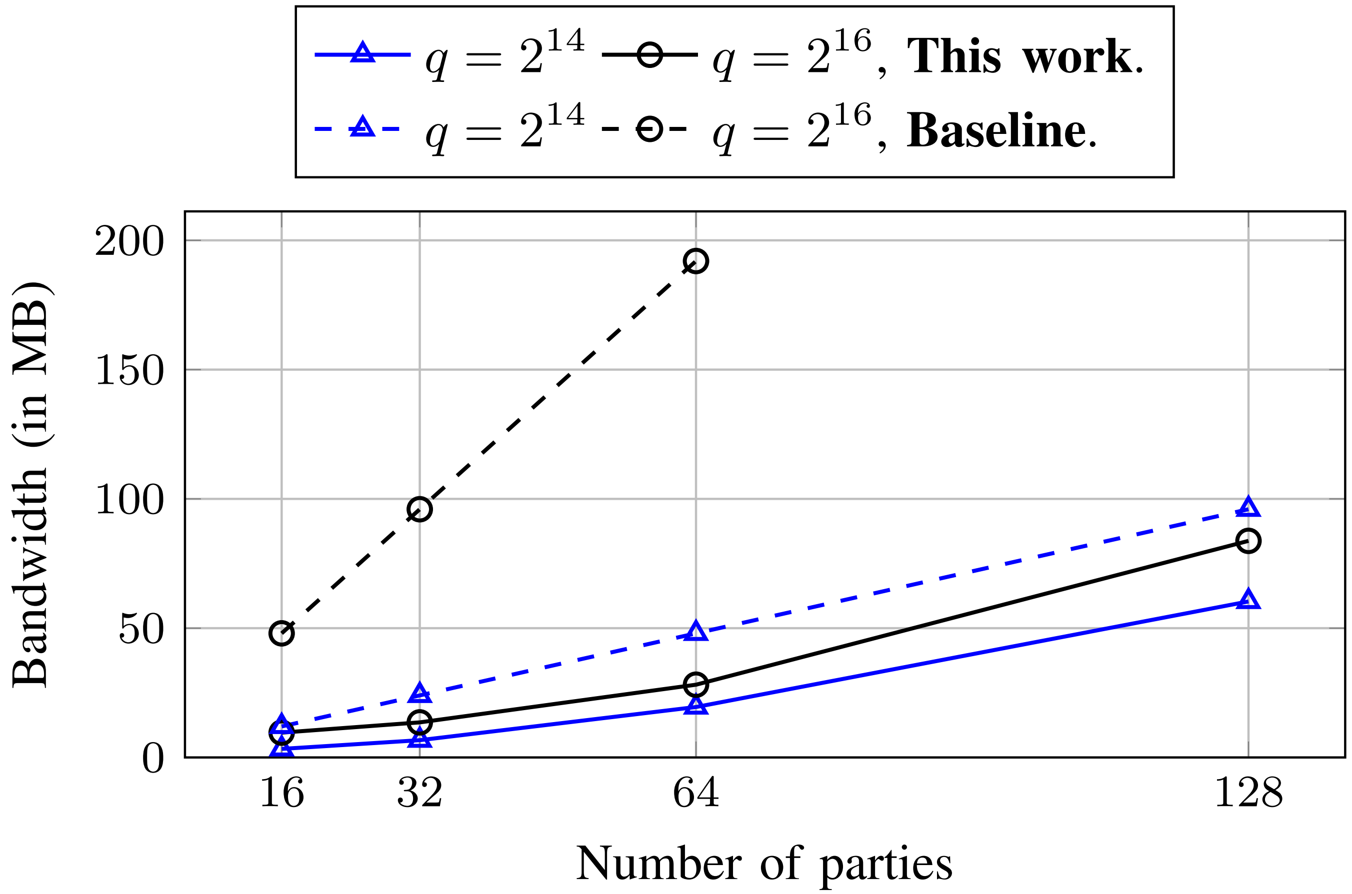

For example, with $q = 2^{16}$, Ours: 13.57 MBytes, Baseline: 96 MBytes (7×)

Summary

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Asynchronous protocol for generating Powers of Tau

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Asynchronous protocol for generating Powers of Tau

Communication Cost (per party)	Computation Cost (per party)	Expected Number of Rounds	Cryptography Assumption
$\mathrm{O}(q + n^2 \log q)$	$\mathrm{O}(q \log n \ \mathbb{G} + n \log q \ \mathbb{P})$	$\mathrm{O}(\log q)$ + ADKG	q-SDH + ADKG

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Thank You (souravd2@Illinois.edu)